National Library of Canada
Bibliothèque nationale du Canada

Canadian Theses Service
Service des thèses canadiennes

Ottawa, Canada
K1A 0N4

## NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30.

## AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

Les documents qui font déjà l'objet d'un droit d'auteur (articles de revue, tests publiés, etc.) ne sont pas microfilmés.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30.

NL-339 (r. 88/04)

Canada

THE UNIVERSITY OF ALBERTA

# COLUMN FLOTATION OF COAL AND FROTH MODEL DEVELOPMENT

by

PHILIP KWADWO OFORI

(C)

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

IN

MINERAL ENGINEERING

DEPARTMENT OF MINING, METALLURGICAL AND PETROLEUM

ENGINEERING

EDMONTON, ALBERTA

SPRING 1988

Permission has been granted to the National Library of Canada to microfilm this thesis and to lend or sell copies of the film.

L'autorisation a été accordée à la Bibliothèque nationale du Canada de microfilmer cette thèse et de prêter ou de vendre des exemplaires du film.

The author (copyright owner) has reserved other publication rights, and neither the thesis nor extensive extracts from it may be printed or otherwise reproduced without his/her written permission.

L'auteur (titulaire du droit d'auteur) se réserve les autres droits de publication; ni la thèse ni de longs extraits de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation écrite.

ISBN 0-315-42884-8

THE UNIVERSITY OF ALBERTA

**RELEASE FORM**

NAME OF AUTHOR PHILIP KWADWO OFORI

TITLE OF THESIS **COLUMN FLOTATION OF COAL AND FROTH MODEL DEVELOPMENT**

DEGREE FOR WHICH THESIS WAS PRESENTED MASTER OF SCIENCE

YEAR THIS DEGREE GRANTED SPRING 1988

Permission is hereby granted to THE UNIVERSITY OF ALBERTA LIBRARY to reproduce single copies of this thesis and to lend or sell such copies for private, scholarly or scientific research purposes only.

The author reserves other publication rights, and neither the thesis nor extensive extracts from it may be printed or otherwise reproduced without the author's written permission.

(SIGNED) .....

PERMANENT ADDRESS:

c/o Mrs Elizabeth Afrakoma

E.J. 51

EJISU-ASHANTI

GHANA, W/AFRICA

DATED APRIL 14 1988

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled **COLUMN FLOTATION OF COAL AND FROTH MODEL DEVELOPMENT** submitted by PHILIP KWADWO OFORI in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE.

...J. Susan C. Finch...
Supervisor

.....

.....

Date...Apr. 12, 1988.....

**ABSTRACT**

On one hand, there is increasing dependence on progressively lower grade ore and coal worldwide due to depletion of high grade material, while on the other hand, demand for higher quality mineral and coal products is on the rise. Conventional technology has come under increasing pressure to resolve this situation. These pressures have also spurred the search for and development of new technology capable of coping with these constraints. An example of these developments is the flotation column. Although widely used in base metal applications its operation is not well understood because of limited design and fundamental operational information.

It is against this background that a laboratory flotation column has been designed and constructed for column flotation studies. A study of the laboratory flotation column has been undertaken, especially its application to coal flotation. The steady state transport behaviour in the recovery zone of the column has been analysed.

After a thorough literature review and the study of the behaviour of the laboratory flotation column froth, a model of the flotation froth has been proposed. Particle, bubble and water transport in the froth have been considered. The model has been verified for coal flotation through the simulation of a number of effects. This work is expected to increase the understanding of processes occurring in the

froth which have not received much attention in the past but are known to contribute to the overall flotation performance. Another application of the model is to provide a basis of rational interpretation of flotation froth behaviour and to enhance the use of pulp phase flotation models, that is, to form the basis of a froth model that can be interfaced with existing pulp phase models to give a complete column flotation model. Due to the complexity of the flotation froth, the proposed model does not completely account for all the mechanisms of mass transfer and further study is required.

## ACKNOWLEDGEMENTS

This work could not have been completed without the help of a large number of people.

I am especially indebted to Professor B.C. Flintoff, my supervisor, for his keen interest, enthusiasm and kind support throughout this work. I also thank Professor L.R. Plitt for his interest, advice and support. The departmental academic and administrative staff have shown tremendous support which I wish to acknowledge.

My thanks also go to Mr. R. Konzuk for his interest and effort in the design and construction of the flotation column. Mr. R. Stefaniuk was very helpful in the electrical installations on the column. Mr. Doug Booth and Jacques Gibeau were helpful with the computing aspect of the work.

Invaluable help was offered by Mr. Bashir Mohamedbhai throughout the experimental processes. I am also thankful for the support of my colleagues in Room 280E.

My thanks also go to Mr. Diego Romero of Line Creek Mines for providing the coal sample used for the study.

Special thanks to Mrs. Mubina Mohamedbhai for typing the script.

Private communication with Dr. G. S. Dobby of University of Toronto and Mr D. G. Foot Jr. of U. S. Bureau of Mines is gratefully acknowledged.

The financial support of the National Science and Engineering Research Council, Ottawa, is gratefully acknowledged.

## Table of Contents


| Chapter | | Page |
|---|---|---|
| 1. | INTRODUCTION AND OBJECTIVES | 1 |
| | 1.1 Flotation Column | 4 |
| | 1.2 Areas of Application of the Flotation Column | 5 |
| | 1.3 Objectives | 7 |
| 2. | FLOTATION PROCESS ANALYSIS | 9 |
| | 2.1 Process Analysis | 9 |
| | 2.2 The Flotation Process | 10 |
| | 2.3 Flotation Modelling Review | 12 |
| | 2.3.1 Classification of Models | 12 |
| | 2.3.1.1 Empirical Models | 13 |
| | 2.3.1.2 Phenomenological Models | 13 |
| | 2.3.1.3 Kinetic Models | 15 |
| | 2.3.2 Other Flotation Pulp Models | 17 |
| | 2.3.3 Distribution of Flotation Rate | 18 |
| | 2.3.4 Hydrodynamic Models | 20 |
| | 2.4 Current Status of Column Flotation Modelling | 22 |
| | 2.5 Areas Requiring More Study | 25 |
| | 2.6 Residence Time Distribution Theory and Modelling Options | 27 |
| | 2.6.1 Residence Time Distribution Theory | 27 |
| | 2.6.2 Modelling Options | 28 |
| | 2.6.2.1 Axial Dispersion Model | 30 |
| | 2.6.2.2 Tanks In Series Model | 30 |
| | 2.6.2.3 Fractional Tubularity Model | 31 |
| | 2.6.2.4 Other Models | 32 |
| | 2.7 Froth Modelling Review | 32 |
| | 2.7.1 Froth Only Models | 35 |

2.8 Foam Models .....37

3. PROCESS EQUIPMENT AND INSTRUMENTATION .....40

3.1 General Description of Equipment Set-up .....40

3.1.1 Detailed Description of the Equipment .....43

4. FLOW CHARACTERISTICS AND RESIDENCE TIME DISTRIBUTION .....49

4.1 Flow Visualization Studies .....49

4.1.1 Justification of the work .....49

4.1.2 Experimental design and procedure .....50

4.1.3 Results and Observations .....52

4.1.4 Conclusions and Recommendations .....54

4.2 Backmixing .....55

4.2.1 Determination of Backmixing .....56

4.2.2 Observations and conclusions .....56

4.3 Residence Time Distribution Studies .....59

4.3.1 Justification of the work .....59

4.3.2 Experimental Design and Procedures .....60

4.3.2.1 The Tracer .....60

4.3.2.2 The tests .....61

4.3.2.3 Conductivity Probe .....61

4.3.3 Tracer Data Processing .....64

4.3.3.1 Normalization of Data .....65

4.3.4 RTD Models Studied .....65

4.3.4.1 Modelling with Series of Mixers ...72

4.3.4.2 Fractional Tubularity Model .....73

4.3.4.3 Axial Dispersion Models .....76

4.3.5 Role of Models in Calculating Flotation Kinetic Parameters .....79

4.3.6 Results and Discussion .....82
5. MODELLING THE CLEANING ZONE .....87
5.1 Justification of The Work .....87
5.2 Model Development .....88
5.2.1 Elements of a Conceptual Froth Model .....89
5.2.2 Conceptual Froth Model .....91
5.2.2.1 Boundary Conditions .....94
5.3 Hydraulic Model .....97
5.4 Solution of Model .....103
5.5 Experimental Design and Procedure .....105
5.5.1 Equipment: .....105
5.5.2 Sample Preparation .....105
5.5.3 Test Procedures .....108
5.5.3.1 Sampling .....110
5.5.4 Results and Discussions .....111
5.6 Model Verification .....124
6. SUMMARY AND CONCLUSIONS .....133
Bibliography .....137
Appendix A Feed and Wash Water Flow Meter Calibration ...143
Appendix B Experimental Data For Runs 1 To 14 .....147
Appendix C Computer Program for Obtaining Model Solution 162

## List of Tables


| Table | | Page |
|---|---|---|
| 5.1 | Bulk Feed Sample Characteristics | 106 |
| 5.2 | Experimental Design for Runs 1 to 18 | 109 |
| 5.3 | Column Flotation Results for Run 15 | 112 |
| 5.4 | Column Flotation Results for Run 16 | 113 |
| 5.5 | Column Flotation Results for Run 17 | 114 |
| 5.6 | Column Flotation Results for Run 18 | 115 |
| 6.1 | Column Flotation Results for Run 1 | 148 |
| 6.2 | Column Flotation Results for Run 2 | 149 |
| 6.3 | Column Flotation Results for Run 3 | 150 |
| 6.4 | Column Flotation Results for Run 4 | 151 |
| 6.5 | Column Flotation Results for Run 5 | 152 |
| 6.6 | Column Flotation Results for Run 6 | 153 |
| 6.7 | Column Flotation Results for Run 7 | 154 |
| 6.8 | Column Flotation Results for Run 8 | 155 |
| 6.9 | Column Flotation Results for Run 9 | 156 |
| 6.10 | Column Flotation Results for Run 10 | 157 |
| 6.11 | Column Flotation Results for Run 11 | 158 |
| 6.12 | Column Flotation Results for Run 12 | 159 |
| 6.13 | Column Flotation Results for Run 13 | 160 |
| 6.14 | Column Flotation Results for Run 14 | 161 |

## List of Figures


| Figure | | Page |
|---|---|---|
| 1.1 | Schematic Diagram of the Flotation Cell | 2 |
| 1.2 | Mechanistic Diagram of Particle Transfer From Pulp to Froth and From Froth to Pulp. (Symbolism Defined on Page 14) | 3 |
| 1.3 | Schematic Diagram of the Flotation Column | 6 |
| 2.1 | Curves Showing Various Extents of Mixing (Levenpiel [1972]) | 29 |
| 3.1 | Schematic Diagram of the Flotation Column Test Rig Including the Major Pieces of Process Equipment and Measuring Devices | 41 |
| 3.2 | Flotation Column Showing Pertinent Measurements and Details of the Feed Port and Sparger | 45 |
| 4.1 | Double Vortex at the Column Feed Intake | 53 |
| 4.2 | Variation of Distance Upstream with Tracer Concentration in Backmixing Tests | 57 |
| 4.3 | Impulse Response Method of RTD Studies | 62 |
| 4.4 | Conductivity Probe Used in RTD Tests | 63 |
| 4.5 | RTD Curves of the Original Data for Various Air flow Rates and 5.5 l/min Feed Flow Rate | 66 |
| 4.6 | RTD Curves of the Original Data for Various Air Flow Rates and 8.0 l/min Feed Flow Rate | 67 |
| 4.7 | RTD Curves of the Original Data for Various Air Flow Rates and 12.2 l/min Feed Flow Rate | 68 |
| 4.8 | Normalized RTD Curves for various Air Flow Rates and 5.5 l/min Feed Rate | 69 |
| 4.9 | Normalized RTD Curves for Various Air Flow Rates and 8.0 l/min Feed Rate | 70 |
| 4.10 | Normalized RTD Curves for Various Air Flow Rates and 12.2 l/min Feed Rate | 71 |
| 4.11 | Comparison of Predicted and Observed RTD from the Various Models for a Feed Flow Rate of 5.5 l/min. | 74 |

Figure Page

4.12 Comparison of the Predicted RTD Profile of the Tanks in Series Model with the Observed Profile .....75

4.13 Comparison of the Predicted RTD Profile of the Fractional Tubularity Model with the Observed Profile for No Air and 5.5 l/min Feed Flow Rate .....77

4.14 Comparison of the Predicted RTD Profile of the Axial Dispersion Models with the Observed Profile for no Air and 5.5 l/min. Feed Flow Rate .....80

4.15 Effect of Air Flow Rate on the Peclet Number .....84

4.16 Effect of Air Flow Rate on the Mean Residence Time .....85

5.1 Conceptual Model of the Froth Zone .....92

5.2 Mass Transfer in an Infinitesimal Slice of the Flotation Froth .....92

5.3 Interstitial-Film Water Transfer in an Infinitesimal Slice of the Flotation Froth .....102

5.4 Schematic Diagram of the Vacuum Sampling Arrangement .....107

5.5 Effect of Wash Water Addition Rate on Product Ash .....120

5.6 Effect of Wash Water Addition Rate on Product Yield .....121

5.7 Effect of Wash Water Addition Rate on Product Percent Solids .....123

5.8 Effect of Wash Water on the Mineral Matter Concentration Profile in the Froth .....125

5.9 Simulated Effect of Wash Water on the Mineral Matter Concentration Profile in the Froth .....126

5.10 Comparison of Simulated and Observed Mineral Matter Concentration Profile in the Froth for no Wash water .....128

Figure Page

5.11 Comparison of Simulated and Observed Mineral Matter Concentration Profile in the Froth for 0.75 l/min. Wash water .....129

5.12 Comparison of Simulated and Observed Mineral Matter Concentration Profile in the Froth for 1.5 l/min Wash water .....130

5.13 Comparison of Simulated and Observed Percent Solids Profile in the Froth for no Wash water .....131

6.1 Magnetic Flow Meter Calibration Curve .....145

6.2 Wash Water Flow Meter Calibration Curve .....146

## List of Plates


| Plate | | Page |
|---|---|---|
| 3.1 | A Photograph of the Test Rig | 42 |
| 3.2 | A Photograph of the Instrument Panel of the Test Rig | 48 |

# 1. INTRODUCTION AND OBJECTIVES

Froth flotation has been established as an effective process for separating fine mineral particles from gangue material. Currently, froth flotation is the most effective and most economical method for recovering fine coal (for example -200 mμ) and this situation is not expected to change in the near future. Factors such as depletion of resources, more stringent environmental control regulations and increasing demand for strategic minerals and alternative energy sources make it essential that more effective methods of fine coal concentration be found. This is expected to happen either by new developments or modification of existing technology. The amount of fine coal and other minerals that require treatment has increased. This is especially true of Western Canadian coals that have a high proportion of fines due to their friable nature.

Flotation is commonly carried out in mechanical cells. Figure 1.1 shows a schematic diagram of the flotation cell. The two mechanisms for particle transfer from the pulp to the froth are by bubble attachment and by entrainment. Particles are lost from froth to pulp by detachment and drainage. Figure 1.2 shows these mechanisms. In the stirred tank flotation cells, particles are kept in suspension by impellers which also generate the bubbles and thus promote particle bubble collision and attachment. This same turbulence that promotes particle-bubble attachment also incites the detrimental effect of detachment of attached

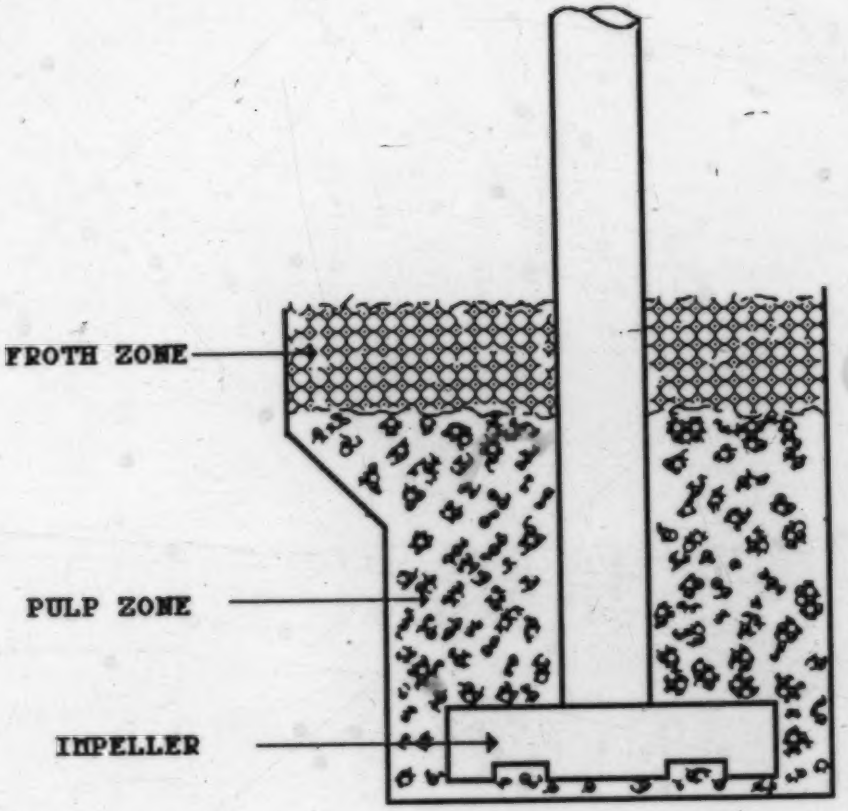


Figure 1.1 Schematic Diagram of the Flotation Cell

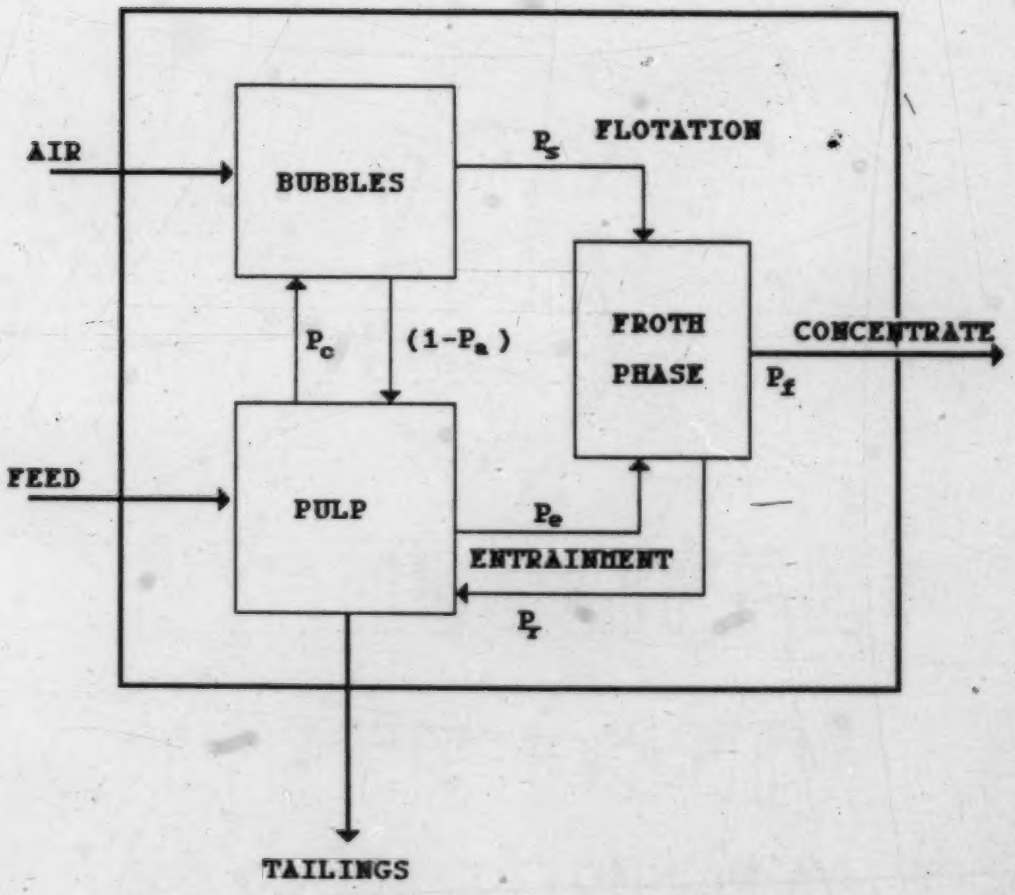


Figure 1.2 Mechanistic Diagram of Particle Transfer From Pulp to Froth and From Froth to Pulp. (Symbolism Defined on Page 14)

particles. The design features of these cells are therefore found to limit the overall efficiency of flotation treatment of fine particles (Moon [1982] & Sastry *et al.* [1982]).

In recent years, attempts have been made to utilize countercurrent flotation columns to increase the efficiency of the flotation process.

### 1.1 Flotation Column

The countercurrent flotation machine was developed in Canada by Boutin, Tremblay and Wheeler some twenty years ago. Its development is described by Anon. (1965), Wheeler (1966), and P. Boutin and D.A. Wheeler (1967).

The flotation column operates on a countercurrent principle in which bubbles generated continuously at the bottom of the column rise through the downward flowing slurry. Column flotation is a continuous process. The physico-chemical principles of bubble-mineral attachment in the flotation column are essentiallly the same as in the conventional cell. What is different is the hydrodynamic conditions; that is, the agitation necessary for particle suspension, which also leads to detachment is not present in flotation columns. Two distinct zones can be identified: a collection zone and a froth zone. The feed enters at a position about two-thirds the column height and decends against rising bubbles generated at the gas sparger. The region between the point at which the feed enters and the position of the sparger roughly defines the collection zone.

The cleaning or froth zone is the region above the collection zone where counter flowing wash water cleans the rising mineralized bubbles. Figure 2.3 shows a schematic diagram of the flotation column.

There are many potential advantages of the flotation column over the conventional cell and these have been elaborately presented by Moon (1982). The most crucial of these advantages is improved metallurgical performance in terms of concentrate grade and recovery. Studies carried out so far seem to indicate that flotation columns have a definite potential of yielding efficient separation of fine particles at high recoveries (Narashimhan [1972], Mathieu [1972], Moon *et al.* [1982], McKay [1986]). Most of these studies have been directed at specific mineral systems.

## 1.2 Areas of Application of the Flotation Column

In North America, research and development of the flotation column is being undertaken in a number of research institutions and universities as well as in some mines. The pioneering work in this area was done by Column Flotation Company of Canada, the principals of which are credited with the idea. Other column flotation research centres include McGill University, University of Toronto, University of Alberta, CANMET (Ottawa), University of California, Berkeley and the U.S. Bureau of Mines research station at Salt Lake City. The mineral systems that have been studied include iron ore (Annon (1965)), copper (Coffin *et al.* [1982]),

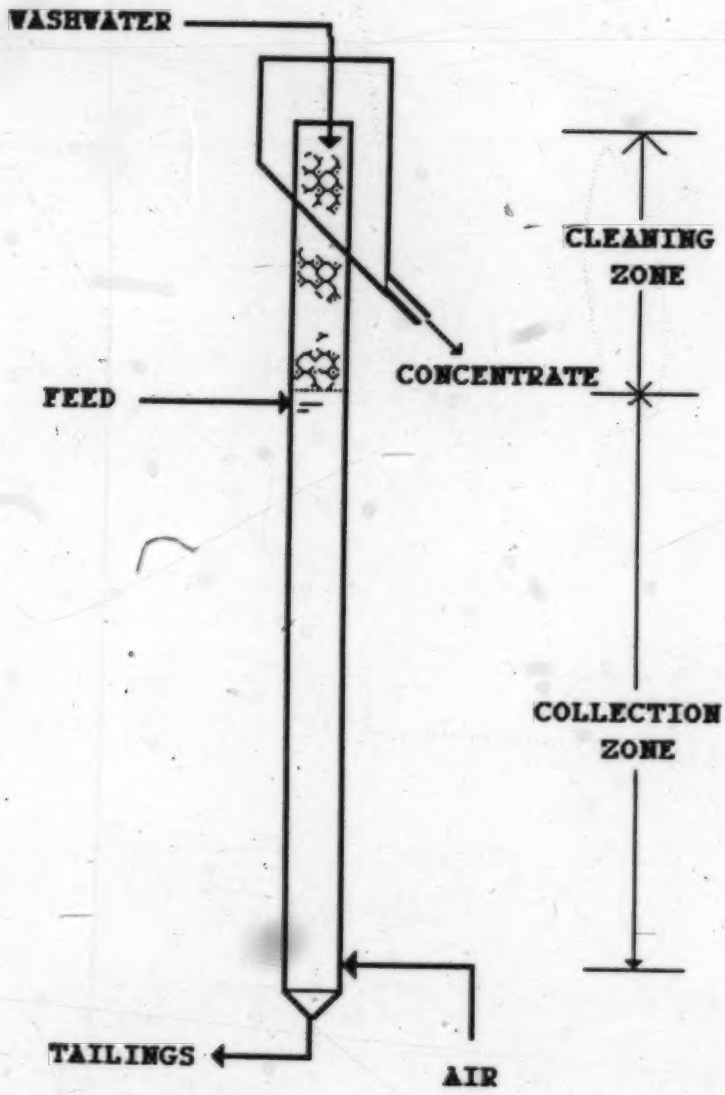


Figure 1.3 Schematic Diagram of the Flotation Column

molybdenum (Mathieu [1972]), chromite and flourite ore (McKay *et al.* [1986]) and coal (Moon *et al.* [1982]).

In Canada, most of the research has focussed on sulphide minerals, particularly copper and molybdenum. Studies involving coal have been limited.

A number of industrial installations exist in North America. These include three columns in the molybdenum cleaner circuit of Mines Gaspe, Quebec, and those installed in British Columbia for copper and molybdenum cleaning, including Gibraltar, Lornex, Highmont and Island Copper (Dobby and Finch [Jan, 1985]). Most reports indicate that columns yield higher grade concentrate with equal or higher recoveries compared to the conventional cells.

In spite of the reported activities and process improvement, the acceptance of the flotation column in the mining industry, though on the upward trend, has been rather slow. This slow acceptance has been attributed to the limited design and operational information. A fundamental understanding of the column flotation process is definitely essential before it can be confidently accepted.

## 1.3 Objectives

To be able to exploit the high potential of separation improvements offered by the flotation columns, a fundamental understanding of the countercurrent flotation process is essential. To understand the process, an analysis of the flotation column is undertaken. The main areas of

investigation are the flow pattern of air and water in the column using direct flow visualization techniques and the residence time distribution, employing the impulse-response technique. In continuous flow operations, such as in the flotation column, the effective rate constant is known to be lowered by extensive longitudinal dispersion. It is therefore important that the mixing characteristics be known.

One mechanism that has been studied in detail is the mass transfer in the flotation column froths which is known to play an important part in the overall flotation process. It was felt that a critical study of column flotation froth and the development of a realistic model that incorporates mass transfer as well as the operating variables would increase the understanding of the column flotation process.

Of course, before any of these studies can take place a flotation column is necessary so the first part of the work was dedicated to the design and construction of a laboratory column. The second chapter gives a brief review of flotation process analysis and modelling. Chapter three describes the laboratory flotation column and the associated equipment. Flow visualization studies of the laboratory flotation column and the residence time distribution studies are outlined in chapter four. Chapter five deals with the proposed froth model and experimental verification of the model. Chapter six presents the conclusion and some recommendations for future work.

# 2. FLOTATION PROCESS ANALYSIS

## 2.1 Process Analysis

Process evaluation and organization require a good understanding of the factors that influence the performance of the process. These factors can be studied using a small scale replica of the process and effecting changes in the input variables while observing how the process responds. This technique can be expensive and take time to to carry out (Himmelblau and Bischoff [1968]). A good approach is to use the study of the replica if it is available, to formulate conceptual representations or models of the process. Mathematical relationships between the process variables are obtained and can be used for computational purposes to assess the performance of the real process. This offers tremendous cost and time incentive and has, in recent years, attracted the attention of mineral process engineers as an important tool for process design and analysis. It must be pointed out that mathematical models of complex processes such as froth flotation rarely represent all the features of the real process if they are to be tractable. Nevertheless, they are extremely valuable tools in simulation, which is the study of a system or parts of it, by manipulation of the mathematical models. The more realistic the models the better the predictions from the simulation study.

Some of the benefits of simulation include the speed and extent of analysis and low cost. Also, operations outside the normal operating ranges can be investigated without the undesirable or disastrous upsets that may occur with the real process. Some of the modern computer process control algorithms use mathematical models.

## 2.2 The Flotation Process

Flotation is a process for separating valuable minerals (usually solids) from waste by utilizing the differences in surface properties of the materials. Minerals which are naturally hydrophobic or can be treated with surfactants to make their surfaces hydrophobic are separated from those which are hydrophilic in a pulp by processes that utilize this surface property difference. For effective separation, the valuable minerals must be liberated by comminution. The mixture of particles (feed solids) must be suspended in water to form a pulp and this state of suspension must be maintained. When air is introduced into the mixture, the hydrophobic minerals or particles adhere to bubbles and rise to the surface where they are removed as mineral laden froth. Various chemicals are added to enhance separation.

Two distinct zones are formed during the flotation process, a pulp zone and a froth zone, each of which plays a significant role in the process. The efficiency of the process is dependent upon several factors that can be grouped into three classes (Dowling *et al.* [1984]): chemical

factors which include reagent addition; equipment factors such as cell design, agitation and froth removal; and operational factors such as feed rate, mineralogy, particle size, pulp density, air flow rate and froth removal.

Flotation has traditionally been carried out in mechanical cells. More recently, flotation columns have been used in cleaner flotation applications. As a result of the radically different design configuration of this equipment, there is a remarkable difference in the hydrodynamic conditions existing in them. While particles are kept in suspension by agitation in the conventional cell, particles fall by gravity in the column in which the feed is introduced in the upper part. Another difference is the fact that air generation in the column is through a sparger while in the mechanical cell it is by the action of the impeller blades in the stator. Froth removal is on all sides of the column, while it is on one or two sides of the cell.

## 2.3 Flotation Modelling Review

The fundamental relationships governing the flotation process are complex and there has been considerable effort to describe these processes by mathematical models. There is extensive material in the literature relating to flotation models and the approach to the formulation varies widely. Published models range from the types that are purely theoretical to purely empirical. Empirical models can usually be developed fairly quickly but have the disadvantage of being limited in validity to a fairly narrow operating range. In contrast, models derived from a theoretical basis are usually valid over a wider range of operating conditions and therefore provide a better insight into the process characteristics. The disadvantage here is that these theoretical models could be fairly complex and may take considerable time to develop but may pay off once developed because of the wide range of applicability. Flotation models are utilized in process design, analysis, optimization and control.

### 2.3.1 Classification of Models

Lynch *et al.* (1981) have classified flotation models into three categories: a) empirical models, b) probability models, and c) kinetic models. Another classification groups the models into empirical and phenomenological models. The phenomenological models encompass Lynch's probability and kinetic models. Other classifications of models in general

have been made by Himmelblau and Bischoff (1968) according to the structure: lumped or distributed, linear or non-linear, steady state or dynamic.

2.3.1.1 Empirical Models

Empirical models are obtained by measuring the response of dependent variables such as recovery or grade when the independent variables (factors) such as collector level or air flow rate are varied. Statistical correlations between the response and disturbance variables are then obtained by regression methods. These are essentially steady state models and should be developed from orthogonal experimental designs such as the two-level factorial design. It is usually risky to extrapolate the results of empirical models outside the range of factors used in the study. Details of publications will not be discussed here.

2.3.1.2 Phenomenological Models

Probability Concept

Schuhmann (1942) proposed the probability model in 1942 and suggested that the recovery rate of particles was related to the probability of success of a sequence of events which occur in the flotation cell before a particle could reach the concentrate launder. This idea was extended by Tomlinson and Fleming (1965) to include the probability that the particle will remain attached and the probability of detachment. Figure 1.2 shows the mechanistic model.

For a particle of size x, the probability of recovery via flotation $P_x^f$, is given by:

$$P_x^f = P_c . P_a . P_s . P_f \qquad [2.1]$$

The probability of recovery via mechanical entraiment $P_x^e$ is given by:

$$P_x^e = (1-P_a) . P_e . P_f \qquad [2.2]$$

where

$P_c$ = probability of collision,

$P_a$ = probability of adhesion,

$P_s$ = probability that the particle will remain attached and be carried into the froth,

$P_e$ = probability of entrainment,

$P_f$ = probability that the particle will survive secondary cleaning in the froth and report to the concentrate.

By reasoning that the mass fraction of material of a component remaining in the pulp after passing through N equivalent cells would be related to the mass of the component in the feed, Kelsall (1961) extended this concept to give, for small P and large N:

$$\frac{W}{W_0} = (1-P)^N$$

$$= \exp(-NP) \qquad [2.3]$$

where

W is the mass of component remaining,

$W_0$ is the mass of the component in the feed,

P is the probability of flotation of the species.

By analogy with chemical kinetics, N can be thought of as the flotation time (t) and P the rate constant (k). The recovery (R) of a species can therefore be written as:

$$R = 1 - \exp(-kt) \qquad [2.4]$$

This equation suggests that at very large flotation times all of a mineral component is recoverable, which is usually not the case. There is usually a limiting recovery $R_{max}$ so that equation 2.4 may be rewritten as:

$$R = R_{max}(1 - \exp(-kt)) \qquad [2.5]$$

2.3.1.3 Kinetic Models

For kinetic model development, the overall mass balance of each flotation system component is used (Imaizumi and Inoue 1963).

The balance can be written as:

$$\frac{dM_i}{dt} = F_i - T_i - C_i \qquad [2.6]$$

in which

$F_i$ = mass flow rate of component i in the feed stream,

$T_i$ = mass flow rate of component i in the tailings stream,

$C_i$ = mass flow rate of component i in the concentrate stream,

$dM_i/dt$ = accumulation of component i in cell holdup.

In a typical semi-batch cell no tailings exit and no feed enters, but both solids and water are continuously recovered in the concentrate. Therefore $F_i = T_i = 0$. Then equation (2.5) becomes:

$$\frac{dM_i}{dt} = - C_i \qquad [2.7]$$

By analogy with chemical kinetics, if the recovery of a component is related to the mass of the component in the cell, a simple rate equation for one component may be written as:

$$\frac{dW}{dt} = -K_n W^n \qquad [2.8]$$

where $K_n$ = flotation rate constant,

$W$ = mass of component in the cell at time t,

$n$ = order of reaction.

For first order kinetics and initial concentration $W_0$, equation (2.7) can be integrated to yield:

$$W = W_0 \exp(-kt) \qquad [2.9]$$

This equation assumes plug flow characteristics and is of the same form as the probability model of equation 2.3.

### 2.3.2 Other Flotation Pulp Models

Several models have been proposed and used to describe the pulp to concentrate material transfer, including those resulting from the chemical kinetic analogy for plug flow semi-batch cells. For continuous flotation systems Imaizumi and Inoue (1963) represented the system by an expression analogous to the equation of a series of fully mixed reactors:

$$\frac{W}{W_0} = \frac{1}{(1+k\tau)^N} \qquad [2.10]$$

where $W$ = mass of component in cell holdup at time t,

$W_0$ = mass of component in cell holdup at t=0,

$k$ = rate constant,

N = number of cells in series,

$\tau$ = mean residence time.

The recovery, R, can then be written as:

$$R = R_{max}\left[1 - \frac{1}{(1+k\tau)^N}\right] \qquad [2.11]$$

Klimpel (1980) has used the following first order model:

$$R = R_{max}\left[1 - \frac{1}{(k\ t)}(1 - \exp(-kt))\right] \qquad [2.12]$$

Arbiter (1951) and Bull (1966) proposed a two parameter second order model in slightly different forms which can generally be written as:

$$R = \frac{R_{max}^2\ k\ t}{1 + R_{max}\ k\ t} \qquad [2.13]$$

Several other models have been proposed and used to describe the flotation process. A review of some of these models have been presented by Dowling *et al.* (1984).

### 2.3.3 Distribution of Flotation Rate

In many cases particles in a cell float at different rates. The presence of fully liberated particles and composite particles in the pulp influence the distribution

of the rate constants. In the case where the particles possess a continuous range of rate constants, the integrated form of the rate equation becomes (Lynch *et al.* [1981]):

$$W = W_o \int_0^\infty \exp(-kt)\, f(k)\, dk \qquad [2.14]$$

where f represents a continuous distribution of rate constants. The accurate estimation of the distribution of rate constants is difficult. Kelsall (1961) suggested that the approximation could be made that the floatable species could be divided into two components of high and low flotation rate constants.

With this approximation, the rate equation for a semi-batch laboratory cell (that is, the solution to equation 2.14) may be written as:

$$W = W_o \left[ \phi \exp(-k_s t) + (1-\phi) \exp(-k_f t) \right] \qquad [2.15]$$

where

$\phi$ is the fraction of slow floating species,

$k_s$ rate constant for slow floating species,

$k_f$ rate constant for fast floating species.

For a continuous series of cells the recovery model for a perfect mixing system is given by

$$R = R_{max} \left[ 1 - \left[ \frac{\phi}{(1+k_s\tau)^N} + \frac{1-\phi}{(1+k_f\tau)^N} \right] \right] \qquad [2.16]$$

where

N is the number of cells in series,

$\tau$ is the mean residence time in the cells.

### 2.3.4 Hydrodynamic Models

It is widely accepted (Schulze [1984]) that one of the most important elementary steps in flotation is the collision of solid particles with gas bubbles where a thin film, having particular properties is formed between the two. Collision is strongly influenced by the hydrodynamic forces at play. He describes collision efficiency as a quantity characterizing the ability of a bubble to capture particles in a fluid flow field as:

$$E = \frac{E_s(1-G) + G}{(1-G)} \qquad [2.17]$$

where

$$E_s = \frac{3r_p}{r_B} \qquad [2.18]$$

G is a gravity parameter (dimensionless particle settling velocity),

$r_p$ is the particle radius,

$r_B$ is the bubble radius.

There are two general theories describing the process of particle-bubble attachment with modifications by various researchers. The first of these theories proposed by Taggart (1942), in 1927 is the gas precipitation theory. Taggart

observed that gas is deposited on hydrophobic surfaces from solutions saturated with gas. In agitation machines, a region of high pressure is created before the the impeller blade and low pressure behind the blade, producing supersaturation and undersaturation respectively. From the supersaturated liquid the air bubbles might then precipitate on hydrophobic mineral particles. Taggart further postulated that most of the minerals which float are buoyed to the surface by air attached in this way. Evidently, this theory is not applicable to flotation column operations.

Gaudin(1932), proposed the direct encounter hypothesis or collision theory. Based on cinematographic evidence by other researchers and his own theoretical treatment, he criticized the gas precipitation hypothesis. These objections seem to have had a negative impact on the gas precipitation hypothesis because most subsequent reseachers have focussed attention on the direct encounter hypothesis approach.

Sutherland (1948) examined the process of collision between particles and bubbles and developed a relationship between the rate constant and collision hydrodynamics. Considering inviscid flow conditions he obtained the following equation:

$$k = 3r_B\theta \operatorname{sech}^2(3V\tau/4d_p)r_pVN \qquad [2.19]$$

where

$r_B$ is bubble radius,

$r_p$ is particle radius,

V is the relative bubble/particle velocity,

N denotes the number of bubbles per unit volume,

$\tau$ is the induction time,

$\theta$ denotes the proportion of particles retained in the froth after fruitful collision.

Dobby and Finch (1985) proposed a model of particle collision based on direct encounter with bubbles. The basis of the model is that particle collection efficiency is directly proportional to the flotation rate constant. This assertion has been made by Jameson *et al.*(1977) as:

$$k = \frac{1.5 v_g E_k}{d_b} \qquad [2.20]$$

where

$v_g$ is the gas velocity,

$E_k$ is the collection efficiency,

$d_b$ is the bubble diameter.

## 2.4 Current Status of Column Flotation Modelling

The flotation column has generated a great deal of excitement in mineral processing research since its invention some twenty years ago. This is especially true over the last six years. However, most of the work done using such a unit has been centred on ascertaining whether

its performance is superior to the conventional cell {(Mckay *et al*. [1986], Groppo[1986], Anon. [1965], Mathieu[1972], etc.)}. Very few attempts have been made to model the flotation column. Although research efforts made to date have resulted in a significant increase in the industrial use of the flotation column, the general availability of operational and design information lags remarkably behind the applications. Models based on critical analysis of the process is one way of achieving design and operational improvements.

The first model of the flotation column proposed by Sastry and Fuerstenau (1970) and was based on the assumption of axially dispersed plug flow. Solutions for specific limiting cases were obtained. The authors did not treat the cleaning and recovery zones separately but identified air and water phases. Since then a number of models have been proposed, most using the axially dispersed plug flow model. These models, like those described earlier, tended to ignore the froth phase behaviour.

Dobby and Finch (Jan. 1985) described the transport characteristics of the recovery zone of industrial flotation columns by the plug flow dispersion model given by:

$$D \frac{\partial^2 C}{\partial z^2} = u_i \frac{\partial C}{\partial z} + \frac{\partial C}{\partial t} \qquad [2.21]$$

where

D is the axial dispersion coefficient,

$u_i$ is the interstitial velocity of the liquid or particle,

z is the axial distance

t is time.

Following Levenspiel (1972) and Wehner and Wilhelm (1956), they gave the analytical solution for first order reaction as:

$$\frac{W}{W_o} = \frac{4a \exp(P_e/2)}{(1+a)^2 \exp(a P_e/2) - (1-a)^2 \exp(a P_e/2)} \qquad [2.22]$$

where:

$$a = \sqrt{(1 - \frac{4 k \tau}{P_e})} \qquad [2.23]$$

and

$$P_e = \frac{D}{u_i L} \qquad [2.24]$$

$P_e$ is the Peclet number.

An equation for estimating the mean residence time was also given by the authors.

The investigators did not model the cleaning zone but suggested that the final recovery of each component is calculated by assuming a value for recovery in the cleaning zone and interfacing it with the recovery in the collection zone.

Another attempt at modelling the flotation column has been made by Xu (1985). For batch flotation he proposes the usual chemical kinetic type equation while using the axial dispersion model for continuous flotation. The dispersion model has also been used by Rice et al (1974) based on the work of Flint (1971). The most recent model for the flotation column has been presented by Lutterell *et al*. (1987). They modelled the flotation column by describing the flow patterns along the length of the column by a series of sections. Each section was further divided into zones and a mass or volume balance taken for each class of particles present.

**2.5 Areas Requiring More Study**

It is clear from the published column flotation models that the axial dispersion model is very popular among the researchers. The indication is that knowledge of the dispersion in a column is important in assessing its performance. Hence the residence time distribution of the column is one area that needs to be studied. A signal introduced at one point in the column is monitored downstream. The various elements of material which enter a

unit can follow very different paths to discharge. The result is that different elements of the material spend different lengths of time in the process unit. The distribution of these times for the stream of material leaving the unit is called the residence time distribution (RTD). A convenient way of determining the dispersion in these columns is by impulse response technique or determination of a residence time distribution function. The transport properties of material through the processing unit are important factors controlling the performance of the unit.

Another very important observation from the literature on flotation column modelling is the lack of modelling effort for the cleaning zone. It has been recognized that the flotation froth plays an important part in the overall performance of the flotation process (Klassen and Mokrousov [1959]), Cutting *et al.* [1982]). It is known to be responsible for the secondary cleaning that takes place. In the flotation column, the effect is expected to be accentuated by virtue of the greater height of froth that is developed and of the cleaning action of the countercurrent flow of wash water. One would therefore have expected more attention paid to flotation column froth. The only reported work so far has been done by Yianatos *et al.* (1985) who used a modified form of the foam model of Steiner *et al.* (1977) to obtain models for the holdup profile and bubble size distribution. A number of models have been proposed for the

froth zone in conventional cells but these are not expected to be directly applicable to the flotation column froth. The lack of a flotation column froth model prompted the investigation and modelling effort in the froth zone, the results of which are presented elsewhere in this thesis.

## 2.6 Residence Time Distribution Theory and Modelling Options

### 2.6.1 Residence Time Distribution Theory

By definition the RTD, E(t), is such that E(t)dt is the fraction of the feed which stays in the vessel for the duration between t and t+dt (Laguitton [1982], Venkataraman and Fuerstenau [1986]). The distribution is normalized such that

$$\int_0^\infty E(t)\,dt = 1 \qquad [2.25]$$

The mean of this distribution, $\tau$, called the mean residence time is physically related to the volume of material in the equipment and the volumetric throughput Q. Hence:

$$\tau = \frac{V}{Q}$$

$$= \int_0^\infty E(t)\,t\,dt \qquad [2.26]$$

The dispersion of the residence times around the mean value $\tau$ is related to the magnitude of mixing or dispersion within the equipment. It can be quantified by the standard deviation or its variance:

$$\sigma^2 = \int_0^\infty E(t)\,(t-\tau)^2\,dt \qquad [2.27]$$

Two limiting conditions can be identified. If no backmixing occurs, all particles entering at time zero will exit at time $\tau$ which leads to zero variance. This condition is termed *plug flow*. The other extreme is *perfect mixing* which means that the column content is perfectly homogenous and consequently the discharge has exactly the same composition as the column contents. Figure 2.1 shows these two types of behaviour.

Actual flow properties lie between these two limiting conditions. Generally, the variance is controlled by the importance of the mixing action relative to transport by convection. This is expressed by the dimensionless Peclet number, $P_e$, defined previously. When $P_e$ is high, the flow behaviour is close to plug flow and the variance is small; when Pe is small the flow behaviour is close to perfect mixing. This is also shown in Figure 2.1. The Peclet number is a function of the equipment size and the magnitude of the mixing forces.

**2.6.2 Modelling Options**

Many types of models can be used to characterize non-ideal flow within vessels and could be single parameter or multi-parameter in nature. The most frequently used models are the axial dispersion model and the tanks in

The material on this page has been removed
because of unavailability of copyright permission.
The material shows curves of various extents of mixing.
The original source of the material is:
Levenspiel O.
Chemical Reaction Engineering
John Wiley & Sons Inc., pp 264, 1972.

series models.

2.6.2.1 Axial Dispersion Model

The axial dispersion models draw analogy between actual flow and a diffusion process. This has been popular with researchers modelling the flotation column. Axial dispersion can be considered as a superimposition of some degree of backmixing or intermixing on plug flow (Levenspiel [1972]). There is an implicit inclusion of dead time.

The axial dispersion model is usually written as a differential equation as given by Equation 2.21. The Peclet number $P_e$, which gives the ratio of convective transport to dispersive transport in a flow system, is defined by Equation 2.24 where $u_i$, L, D, and are respectively the axial convective velocity, characteristic length (taken to be the length of the recovery zone) and the axial dispersion coefficient.

The Peclet number whose magnitude completely defines the RTD function can be obtained from the expression:

$$\sigma^2 = 2/P_e - 2/P_e^2(1 - e^{-P_e}) \qquad [2.28]$$

The variance is available by solution of equation 2.27.

2.6.2.2 Tanks In Series Model

Besides the dispersion model, the tanks in series model is the other single parameter model that is widely used. Here, the fluid is viewed as flowing through a series of stirred tanks and the single parameter is the number of

cells in the series. If the series of tanks have equivalent holdup volumes the dimensionless RTD function is given by: (Levenspiel[1972])

$$E(\theta) = \frac{N^N \theta^{N-1} \exp(-N\theta)}{(N-1)!} \qquad [2.29]$$

where

$\theta$ is the dimensionless time ($t/\tau$)

For tanks not of equal holdup volumes, the number of model parameters increases.

2.6.2.3 Fractional Tubularity Model

The residence time distribution profile can also be analyzed as a combination of plug flow and perfect mixers in series, often called the fractional tubularity model, (Nauman and Buffham [1983]) characterized by a sharp first appearance time $\tau_p = \theta_p \tau$ and by an exponential tail. $\tau$ is the mean residence time of the system, and $\tau_p$ is the time after which the concentration is greater than zero. The fractional tubularity model is one of the simplest but most useful in residence time distribution theory. It is simple to fit and can readily be incorpolated in kinetic models. This provided the incentive to study it in the present work even though it has more parameters than the series of mixers and axial dispersion models.

#### 2.6.2.4 Other Models

When the one or two parameter models are unable to satisfactorily account for the deviation from plug flow or perfect mixing, more complex models are employed. Examples of these include models for complete mixing with dead space and short circuiting, plug flow with dead space and short circuiting and many others described by Wen *et al*. (1975). Rogers and Gardiner (1979) have used a six parameter finite stage transport model. The details of these models will not be discussed here.

## 2.7 Froth Modelling Review

Not much information exists in the literature on froth modelling (Cutting *et al*. 1982, 1986) despite the fact that it has long been recognized that subprocesses operating in the froth effects some cleaning action (Klassen and Mokrousov [1959], Maksimov and Khainman [1965]). The probability and chemical kinetic approaches of modelling the flotation process did not explicitly take the effect of the froth into consideration. There are physically two distinct sections, the pulp and froth zones with different mass transfer rates which have to be treated as such. In many models the effect of the froth phase is usually lumped together with several other effects into one parameter such as the flotation rate constant. Moys (1978), Cutting *et al*. (1982) have clearly shown the distributed nature of the froth by identifying some of the processes occurring in the

froth, film drainage and water and particle transfers. According to Cutting *et al.* (1984), the principal uncertainties limiting the prediction capabilities of these models are contributed by processes operating in the flotation froth phase which have not been studied in great depth. With the advent of the countercurrent flotation column in which considerable froth heights are developed, the study of the processes occurring in the froth is essential.

Different approaches have been used to model the flotation froth. One approach is to regard the froth zone as a separate region in the flotation system and construct a model for the whole process in which the effect of the froth phase is represented by one or more parameters. This approach has been used by Arbiter and Harris (1962) and extended by Harris and Rimmer (1966). They represented pulp and froth as separate completely mixed phases. The basic assumption of a perfectly mixed froth has been questioned (Cooper 1966, Mika 1967). Lynch *et al.* (1981) also considered the froth phase as a separate perfectly mixed region and obtained a model involving a modified rate constant.

Loveday and Marchand (1972) showed that the return of particles from the froth has the effect of reducing the apparent residence time. King (1970) characterised the perfectly mixed flotation cell operating continuously by a modified perfectly mixed reactor model as:

$$M_T f_T = \frac{M_I f_I}{1+\gamma k \phi(D) A \, S_{av} \theta} \qquad [2.30]$$

where

M = mass flow rate of solids

$\theta$= average residence time

k = the flotation rate constant

A = bubble surface area per unit volume

$S_{av}$ = the fraction of bubble surface area not covered with solids

$\phi(D)$ = multiplying factor to account for variation of flotation constant with size

$\gamma$ = froth transmission coefficient which represents the fraction of solids leaving the pulp phase that finally passes over the froth lip and are recovered in the concentrate,

subscript T refers to the tailing stream and I the feed stream.

The froth transmission coefficient in effect accounts for all the subprocesses occurring in the froth. This serves the same purpose as the froth efficiency factor that Cutting *et al*. (1982) described later, with the important difference that the efficiency factor is found from a model accounting for film and column drainage.

### 2.7.1 Froth Only Models

The second approach to flotation froth modelling is to isolate the froth phase, identify the micro-processes occurring in the froth, and construct models to describe them. These models could be interfaced with the pulp models to describe the entire flotation process. This approach has been followed by some recent researchers which include Moys (1978, 1984), Cutting *et al.* (1980, 1982, 1986), Yianatos *et al.* (1985), Bascur and Herbst(1982) and Bascur (1982).

Bascur and Herbst proposed a four states population balance model for the flotation process; two states for the pulp and two for the froth. The two states for the froth were;
a) attached in the froth and b) free in the froth.
This model considered particle attachment and detachment as well as water transfer. The model has been used as a basis for studying computer control of a pilot plant flotation circuit.

Moys (1978)advanced a froth model based on the countercurrent plug flow assumption by identifying the subprocesses occurring in the froth. He considered bubble-particle attachment in the pulp, particle detachment in the froth, water and particle entrainment and water and particle return from froth to pulp. First order kinetics were assumed and the resulting equations were solved to relate froth height, grade and solids concentration. Most of the microprocesses were taken into account. Attachment of

particles to bubble in the froth was not considered, neither was the liquid holdup in the froth.

Moys (1984) proposed two more models describing the residence time distribution and material transfer in the froth. The first one drew an analogy between mass and heat transfer and described the path of perfectly floatable particles as streamlines represented by a two dimensional Laplace equation. The second model is a two stage conceptual model which simplified the solution of the Laplace equation. Simulations using these models by the author showed that both the vertical and surface velocities have their highest values near the concentrate weir and correlates with the fact that the residence time of the froth near the weir are small and may be responsible for most of the entrained gangue and water entering the froth.

The investigators at the Warren Spring Laboratory have observed (Cutting *et al*. (1982)) that processes operating in the froth phase have not been studied in great depth and that limitations on the use of flotation models are largely contributed by these processes. They identified two drainage mechanisms in production scale flotation froth: 1) film drainage in which water and solids drain around the air bubbles and is present throughout the whole froth, and 2) column drainage in which material descends rapidly in single vertical locations in the froth. They characterized the mass transfer in the froth with a cell efficiency factor which they defined as the probability of transfer from the

froth/pulp interface to the froth discharge lauder. This efficiency factor could be interfaced with the pulp model to yield the overall model for the flotation stage in the general flotation flowsheet.

It appears that so far only one published attempt has been made to describe the flotation column froth by mathematical models. This attempt was made by Yianatos *et al.* (1985). These investigators proposed that the froth consist of three sections: 1) expanded bubble bed, 2) packed bubble bed, and 3) conventional draining froth. The packed bubble bed was described by a slight modification of a model of cellular foams proposed by Steiner *et al.* (1977). From the resulting model, the gas holdup fraction, bubble diameter and film thickness along the froth height could be generated.

The expanded bubble bed was modelled with an Ergun (1952) type equation from which bubble diameter could be correlated with gas holdup fraction.

## 2.8 Foam Models

Considerable work has been done to characterize the behaviour of foams with mathematical models, a few of which will be reviewed here. Most of the early research work has been reviewed by Bikerman (1972). Leonard and Lemlich (1965) presented a theoretical model for interstitial liquid flow in foams by relating the structure of the foam to the physical properties of the surfactant through a differential

momentum balance within the Plateau border. Ho and Prince (1971) obtained a correlation between gas superficial velocity, gas holdup fraction, and volume average bubble diameter based on an analysis of liquid flow through the network of Plateau borders formed at the edges of regular dodecahedral bubbles. This model contains one empirical constant. Desai and Kumar (1982) have modelled the foam as three subsystems; near horizontal and near vertical Plateau borders and bubble films. From the models they generated the gas holdup fraction as a function of the height of foam.

The most promising models of foams in terms of their applicability to flotation froths have been presented by Hartland and Barber (1974) and by Steiner *et al.* (1977) Hartland and Barber represented foams by a structure of pentagonal dodecahedra in which liquid is carried upwards by the films forming the faces of the dodecahedra and returns to the bulk by gravitational flow in the Plateau borders. The model was derived by obtaining the radius of plateau borders from equating the liquid downflow and liquid upflow at steady state. Film thinning was described by Reynolds equation of film drainage from which the film thickness could be computed. A relationship for total liquid holdup was combined with the film drainage and flow in Plateau borders to obtain the liquid holdup as a function of height, physical properties, gas velocity and bubble size.

Steiner *et al.* (1977) have included the effect of foam removal as well as the fact that the bubble surfaces are not

completely rigid. They have also included the effect of bubble coalescence as a stochastic phenomenon. These modifications make the last two models mentioned above promising as possible candidate models for inclusion in flotation froth models. An attempt has been made to incorporate these ideas into the froth model development reported elsewhere in this thesis.

The review of the literature of flotation modelling reveals that while there is extensive material on flotation models as a whole, study of the flotation froth as a separate entity is lacking. Only recently have such studies attracted the attention of researchers such as Moys, Cutting and the Warren Spring group and Yianatos and the McGill group. With the increasing acceptance and use of the flotation column in which considerable heights of froth exist, there is a definite need for froth studies.

# 3. PROCESS EQUIPMENT AND INSTRUMENTATION

## 3.1 General Description of Equipment Set-up

A laboratory flotation column test rig has been installed for use in the coal flotation and froth modelling tests. Flow and mixing characteristics in the column were studied using direct flow visualization techniques and residence time distribution studies.

The circuit consists of a baffled 370 litre capacity conditioning tank in which the feed is conditioned prior to flotation for a semi-continuous operation. After conditioning, the feed is pumped into the flotation column by a variable speed moyno pump. A flow meter on the feed line records the feed flow rate. Wash water is added at the top of the column through a shower head device. Wash water is measured by a rotameter. The concentrate is removed from the top of the column. Air is introduced into the column at the bottom through a porous steel sparger. The tailings are removed from the column by another variable speed moyno pump which is also used to control the pulp/froth interface level. A schematic diagram of the test rig is shown in Figure 3.1. A photograph of the equipment is provided in Plate 3.1.

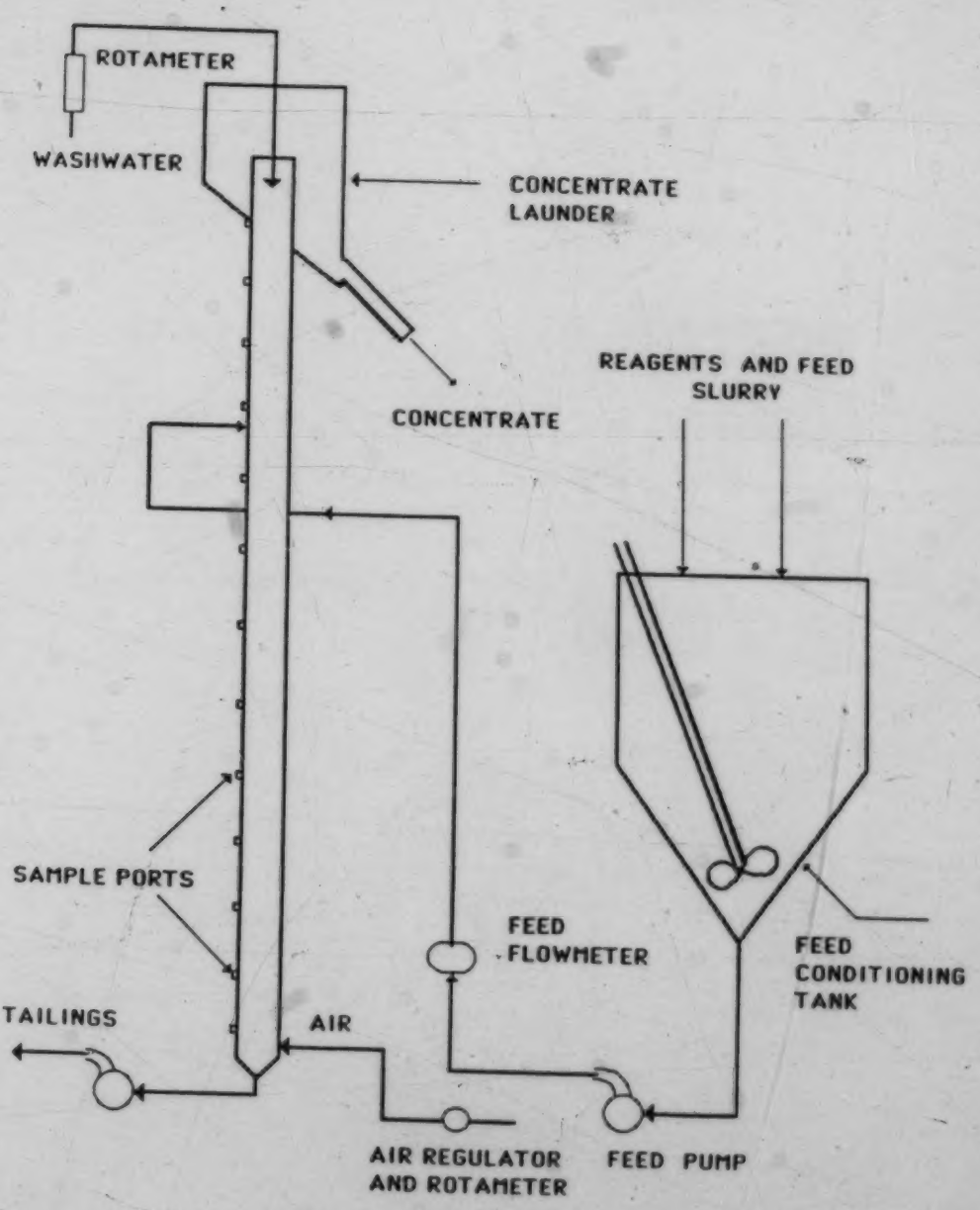


Figure 3.1 Schematic Diagram of the Flotation Column Test Rig Including the Major Pieces of Process Equipment and Measuring Devices

Plate 3.1 A Photograph of the Test Rig

### 3.1.1 Detailed Description of the Equipment

The column

The column is made of transparent lucite sections of 6.35cm inside diameter and 0.64cm wall thickness. The total height is 3.6m and consists of one 1.2 metre section, two 0.6 metre sections and four 0.3 metre sections and are all flanged. The sections are bolted together and sealed with "O" rings. A short conical section with 2.54cm discharge port is attached to the bottom section of the column. This modular design makes it possible for convenient variation of the total height of the column as well as the feed entry point and the froth height.

Sample ports of 0.64cm diameter are provided at 15cm intervals on the column. These are shown in Figure 3.2. Two manometers are installed on the column to measure the air holdup fraction.

The feed port which is a 1.6cm inside diameter lucite tube is situated two-thirds of the length of the column from the bottom. This position could be easily varied by adding or removing some of the sections. The feed port is designed to protrude into the column and discharge at its underside to reduce short-circuiting of feed into the froth and hence into the concentrate as depicted in Figure 3.2. A 15.24cm inside diameter cylindrical overflow launder, also made of lucite, is fitted concentric to the top section of the column. The slope of the bottom is 45 degrees and is provided with a discharge port. The advantages of using

transparent lucite material for the column is that it makes it possible to conduct direct flow visualization study. In the coal flotation tests, the flotation response could be visually observed quickly. Figure 3.2 is a diagram of the column showing pertinent measurements.

Feed Conditioner

The feed conditioning tank is baffled and is equipped with a stirrer to mix the reagents and keep the solid particles in suspension. During the conditioning period a gate valve on the discharge line is kept closed.

Feed and Tailings Pumps

These are positive displacement moyno pumps. A helical shaped stainless steel rotor rotates in a rubber stator to produce the pumping action so that the pumping rate is proportional to the rotational speed of the shaft. The shaft of the feed pump is driven by a 1/3 hp variable speed motor while that of the tailings pump is driven by a 1/4 hp motor. The speed of the motor of the tailings pump is manupulated to control the pulp/froth interface.

Feed Flow Meter

A 0.95cm magnetic flow meter model 10D1475 with a signal converter, supplied by Fischer and Porter, is installed on the feed line to measure the volumetric flow of the feed to the column. The fluid, flowing through the magnetic field of the meter, produces an induced voltage proportional to the

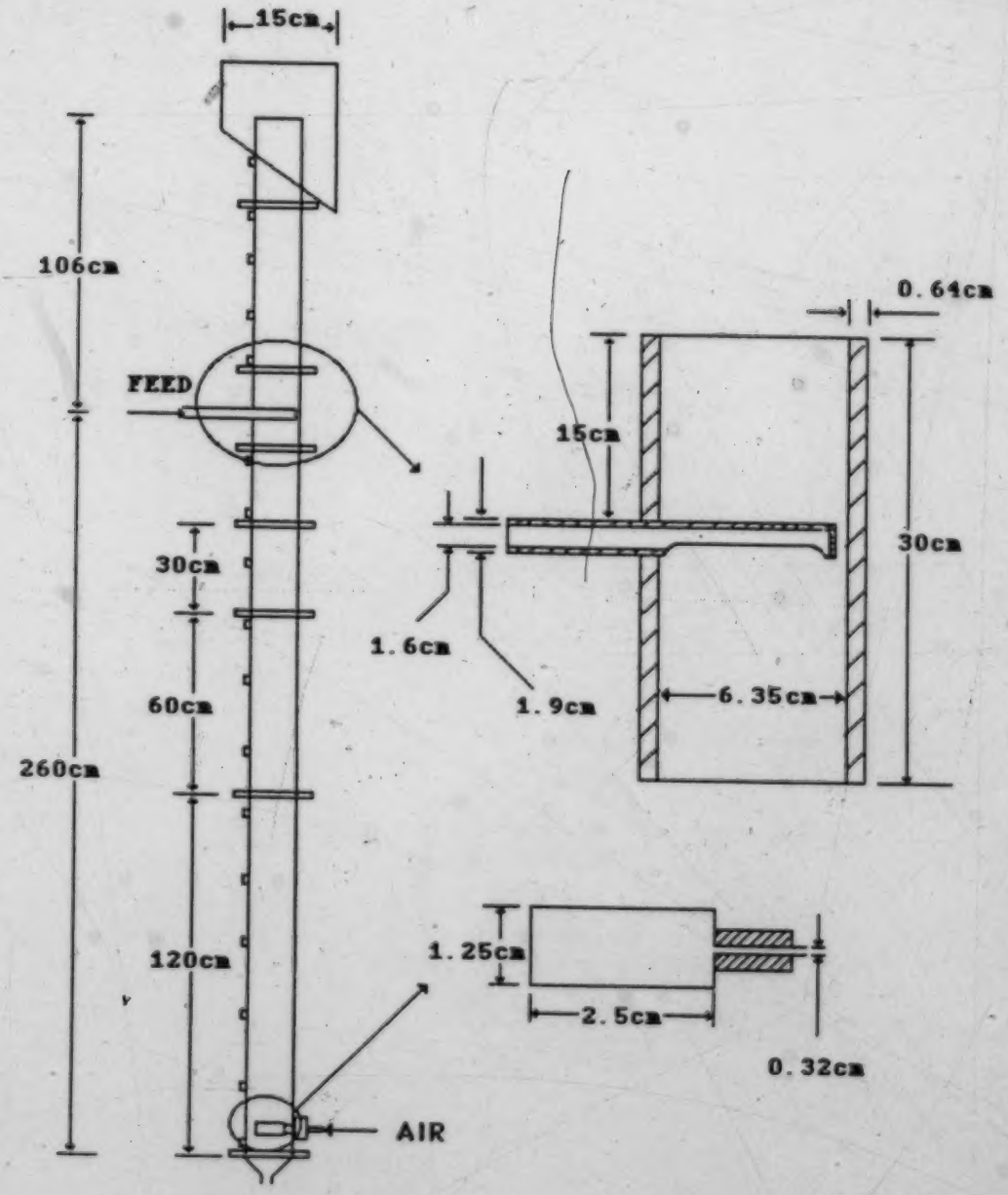


Figure 3.2 Flotation Column Showing Pertinent Measurements and Details of the Feed Port and Sparger

flow rate of the fluid. The induced voltage is processed by the signal converter and the corresponding flow rate in litres per minute is displayed on a digital recorder. The cumulative flow is also recorded. The units of flow rate can be changed by recalibration. The range of measurement is set at 0 to 8 l/min but provision is made to alter this range by appropriately adjusting a variable resistor in the signal converter.

The calibration procedure is given in Appendix A.

Wash water Supply

Wash water is supplied from the laboratory cold water mains. Two gate valves on the line are used to adjust the pressure and flow rate to the desired levels. A Brooks rotameter with a steel float measures the flow rate of the wash water to the top of the column. The wash water is supplied through a distributor situated 5cm below the top of the column. This addition point can be changed. The calibration procedure of the wash water flow meter is also given in appendix A.

Air Supply

Air supply is from the building supply at a pressure of about 100 psi. A pressure regulator is used to reduce the pressure to about 30 psi. The air enters the bottom of a rotameter and flows upwards past a float. The air flow rate in litres per minute is read directly from the position of

the float. From the rotameter the air goes through a 0.32cm tube to a porous steel sparger 2.5cm long and 1.25cm in diameter and of pore size two microns situated at the bottom of the column.

Instrument Panel

A centralized instrument panel is attached to the column support to allow for easy control of column operation. The speed regulators for the feed and tailings pumps are mounted on this panel as are the air and wash water regulators. The rotameters are also mounted on the panel. Also mounted on the panel is the digital readout of the feed flow meter. A photograph of the instrument panel is shown in plate 3.2.

The modular design, the transparent nature of the lucite material and the centralised instrument panel afford the ease of operation and control of the column. Instrumentation for automatic control can easily be added.

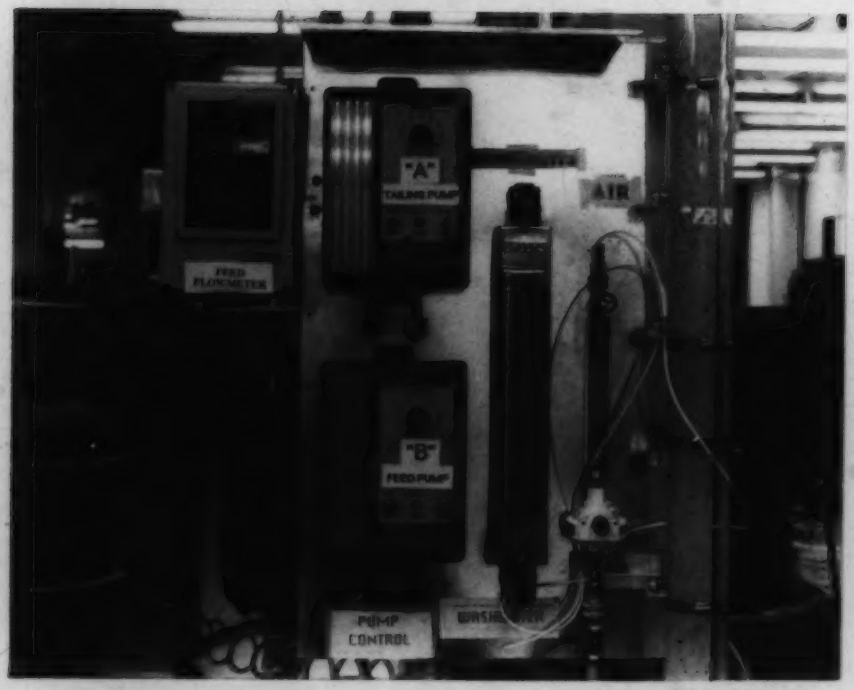


Plate 3.2 A Photograph of the Instrument Panel of the Test Rig

# 4. FLOW CHARACTERISTICS AND RESIDENCE TIME DISTRIBUTION

## 4.1 Flow Visualization Studies

### 4.1.1 Justification of the work

As has been previously mentioned, the flotation column is considered to be potentially more likely to yield better results than the conventional cell in metallurgical performance by virtue of its design and operational principles. However, the acceptance of the flotation column in the mining industry, though on an upward trend, has been rather slow. One of the reasons for this situation is the lack of fundamental understanding of the column. A good understanding of the operational principles and mass transport mechanisms is expected to increase the confidence of the mineral industry in the system.

To be able to understand and possibly improve on the mass transfer occurring in the column, knowledge of the flow field is very important. Direct flow visualization techniques are a strong research tool for monitoring complex flow behaviour. These techniques involve either dye injection or suspended particles for visual observation of the flow pattern.

This section presents some results from the investigations of the flow and mixing processes in a laboratory flotation column. The first part of the experimental work was dedicated to flow visualization in a

single phase and two phase flow in which water and then water and air were used. In both cases a frother, methyl isobutyl carbinol (MIBC), was added to give more characteristic bubble sizes. A dye was injected to obtain some qualitative information on the flow pattern.

Dye particles are very small and thus may reasonably be assumed to have no relative motion in their carrier liquid. Diffusion due to concentration gradients is very slow and may be assumed to be insignificant. The motion of the dye is hence assumed to be the same as the carrier fluid.

In order to achieve adequate contrast to the carrier fluid, the dye concentration must be quite high and be injected such as to cause minimal disturbance to the fluid flow.

Backmixing characteristics were studied using a salt tracer technique.

### 4.1.2 Experimental design and procedure

The principal technique used in visualizing the flow involved the introduction of a dye as tracer material into the flow stream. The tracer material used was food colouring mixed with milk. A dye injection probe consisting of #18 syringe needle was used. Care was taken to ensure that the dye did not enter the test solution in the form of a jet. The operations were conducted in open circuit to avoid contamination of the test fluid with the dye. The test fluid was water to which a frother had been added. MIBC was added

in dosages of 30 micro-litres/litre (30 ppm) and the solution was thoroughly mixed.

Tests were conducted for both single and two-phase flow. In the single phase tests, flow in the recovery zone was examined with particular attention paid to the feed entry point to see if any degree of mixing and turbulence occurred there. Flow in the lower portions of the column was also studied at three different flow rates. For the tests in the feed entry point, the dye was injected into the feed line. At the lower portion of the column the sample ports in that section were used to introduce the dye. In the two-phase tests an attempt was made to observe the nature of the flow pattern when air flows countercurrent to the test fluid.

To learn whether any short-circuiting or entrainment of feed water into the froth existed, visualization of the froth zone at various air flow rates were attempted. The flow rates at which the tests were done were 4.1, 7.2 and 12.2 l/min. In each case the dye was injected using a hypodermic syringe at two points; 104cm and 168cm from the base of the column. Inlet flow rate, wash water and air flow rates were adjusted, and steady state conditions in level attained before the dye injection.

Photography

For recording the actual dynamic flow, a video recorder is without comparison (Chandran *et al.* [1985]). It provides

immediate results without photographic development delays. It allows the study of a sequence of flow events and can also be set to freeze an instant in time. A video recording of the various flow patterns was made using a 3/4" VCR camera and cassette tape deck. The camera was equiped with a zoom lens. A light source was used for illumination. This recording was later played back and analyzed for peculiar flow behaviour. The quality of recording could be determined immediately and reshot if necessary.

### 4.1.3 Results and Observations

Recovery Zone

The feed port of the column is situated two-thirds of the height of the column from the bottom and at right angles to the column as shown in Figure 3.2. This configuration of feed port causes the feed material travelling at high flow rates to impinge on the opposite wall of the column. This results in vibration of the column and more importantly, causes a separation of flow into two symmetrical vortices rotating in opposite directions as illustrated in Figure 4.1. The superposition of this rotating flow on the bulk axial flow results in a fairly weak swirling flow. The swirling flow was revealed by the dye introduced into the lower portion of the flow. The swirl increases with increasing feed rate and gradually decays as it decends along the length of the pipe towards the bottom of the column. Another result of this type of flow is some degree

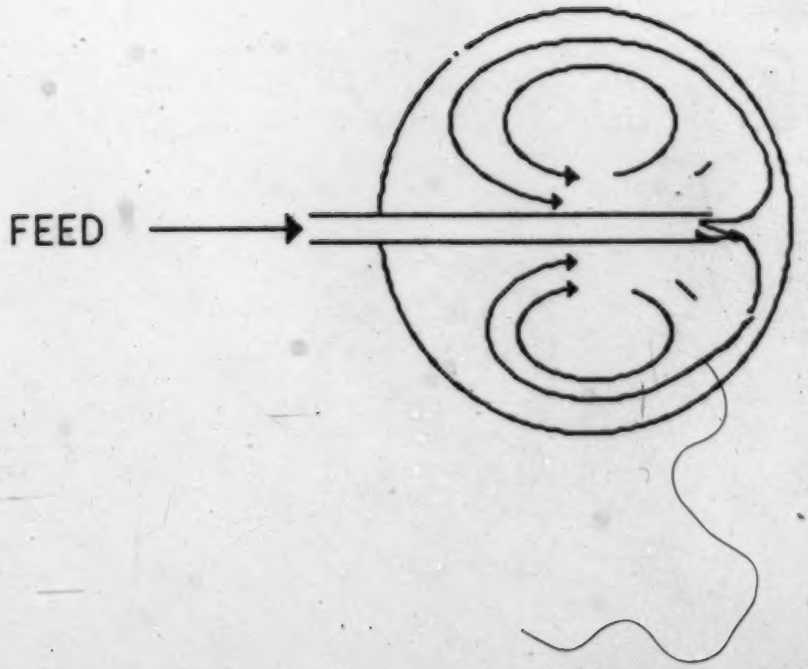


Figure 4.1 Double Vortex at the Column Feed Intake

of mixing in the vicinity of the feed entry point and therefore a distortion of the expected plug flow in the tubular reactor. This flow separation may induce backmixing in the reactor too.

Visualization in the froth zone was not very successful as the particles were difficult to see.Only very little of the dye introduced into the feed line entered the froth zone.

### 4.1.4 Conclusions and Recommendations

This qualitative study of the flow pattern revealed the flow separation and swirl flow which are both detrimental to column performance by the introduction of mixing and backmixing. Swirl flow may result in classification of feed material if it is of sufficiently high intensity, since the larger and heavier particles may be thrown to the outer part of the column by the centrifugal/radial force. A possible design change that would reduce this effect would be to introduce the feed through a larger feed line which is curved downward into the column, but a large tube in the column would itself be an obstruction to mineral laden air bubbles moving upwards. Another disadvantage of a larger feed line would be the tendency for the feed solids to settle out in the feed line due to inadequate carrying velocity. This could be offset by a dual feeding arrangement; gravity feeding at low flow rates of feed and pumping at higher flow rates.

These qualitative tests suggest that some degree of mixing occurs in the column and therefore backmixing tests and residence time distribution tests would give more revealing information about the degree of mixing.

**4.2 Backmixing**

The performance of tubular reactors for multi-phase reactions such as in the flotation column is impaired by the occurrence of unfavourable flow profiles and convective fluctuations. These can lead to large degrees of backmixing which must be suppressed in order to minimize axial fluid dispersion. Backmixing is the result of superposition of radial exchange of fluid elements on the longitudinal or axial fluid flow. The orientation of the feed port in the column seems to influence this, and therefore an investigation of backmixing was considered worth undertaking. A test was therefore run to ascertain the existence and degree of backmixing. Backmixing is also significantly induced by gas flows which cannot be avoided in flotation. It is also known to increase with the diameter of the column (Shah *et al.* [1978]), that is, usually more backmixing occurs in commercial columns than in pilot scale columns.

Backmixing is a flow pattern that is intermediate between the two ideal cases of plug flow and perfect mixing (Mecklenburgh and Hartland [1975])

Backmixing has the property that some material is flowing countercurrent to the main stream and that there is no variation in conditions in a cross-section normal to the main flow. A positive test for backmixing is the detection of tracer upstream of the injection point.

### 4.2.1 Determination of Backmixing

The method employed to study backmixing in the flotation column was to inject a tracer at the exit of the column and then measure the upstream concentration profile. The tracer used was sodium chloride which was injected using a number 18 syringe at a sampling port 15cm from the bottom of the column. The concentrations were monitored at regular intervals of 15cm upstream from the injection point. A conductivity probe described in detail in Section 4.3.2 was used to analyze the response. The response was plotted continuously on a chart recorder.

### 4.2.2 Observations and conclusions

Observation of the response indicated that there was a minimal amount of backmixing when no air was flowing even at the first sampling point 15cm upstream. As expected the concentration of tracer dropped with distance upstream from the injection point as illustrated in Figure 4.2.

Four different air flows were used to determine the effect of air flow on backmixing. The flow rates were; 1.0, 1.5, 2.0, and 2.5 litres per minute. Surprisingly the

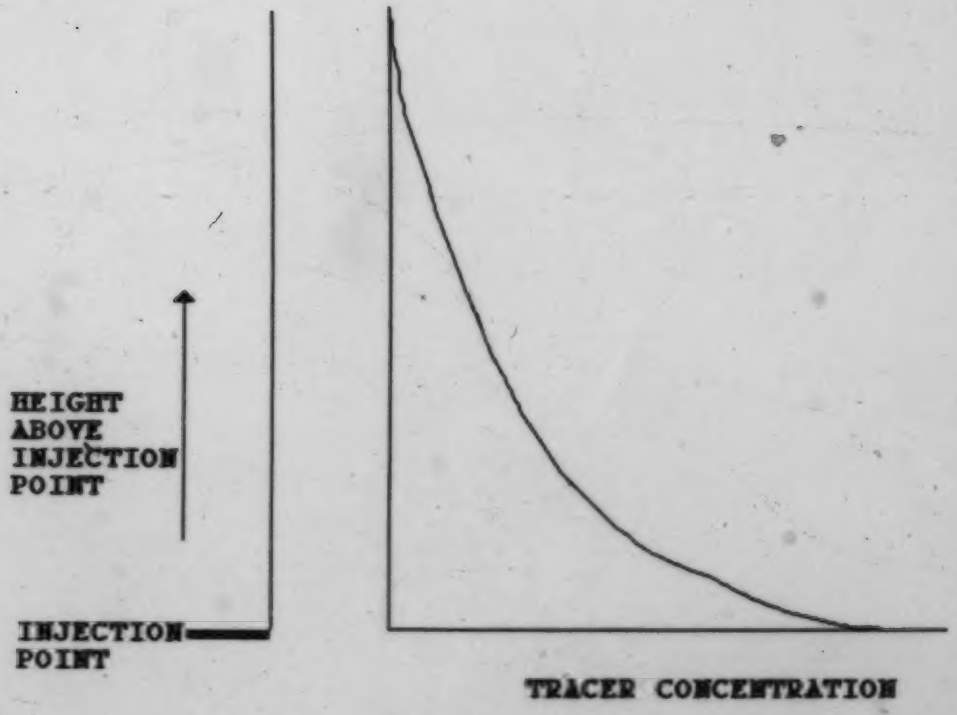


Figure 4.2 Variation of Distance Upstream with Tracer Concentration in Backmixing Tests

upstream tracer concentration dropped with increasing air flow. The initial supposition was that the swirl flow was the cause of the backmixing. The observation of only a small degree of backmixing when there was no air flowing did not support this supposition. A reasonable explanation of the drop in tracer concentration is that channelling that leads to slug flow sets in as the air flow increases. At 1.0 litre per minute air flow rate, the air bubbles are more uniformly distributed in the cross-section of the column. At higher air flows, more bubble coalescence occurs and channelling begins to take place. The result is an uneven distribution of air bubbles with a reduced specific surface area. The amount of tracer associated with the bubbles is reduced with the reduction in bubble surface area. The accuracy of the conductivity sensor may also be affected by non-uniform distribution of air bubbles.

It can be concluded that,

a) some backmixing occurs in the column under normal operating conditions,

b) backmixing in the column at the flow rates studied is primarily induced by the countercurrent flow of air and appears to be significantly influenced by the distribution of the air,

c) The swirl flow seems to play only a minor role in the backmixing.

A residence time distribution study would be appropriate to determine the extent of backmixing.

## 4.3 Residence Time Distribution Studies

### 4.3.1 Justification of the work

The performance of the flotation column with respect to recovery and selectivity depends not only on the intrinsic kinetics of the reactions occurring, but also on the physical rate processes such as interphase mass transfer. In flotation, interphase mass transfer is probably the most important mechanism of separation. The effects of these physical rate processes on flotation performance have been known to depend on the dynamics of the various phases, that is, the shape and length of the path followed by individual elements of the flow as well as the velocity of the elements. The qualitative flow visualization and backmixing tests discussed previously have shown that some backmixing does occur in the column which would result in a distribution of residence times. A thorough understanding of residence time distribution (RTD) concepts and techniques would be of considerable benefit in the design and analysis of flotation columns. This aspect of the work is intended to evaluate the residence time distribution in the flotation column, and the extent of mixing, which is useful information for design purposes.

It is generally recognized that the flow pattern existing in a piece of equipment lies somewhere between the two ideal flow patterns of plug flow and perfectly mixed flow. The average statistical parameters are usually

determined by the impulse response technique in which a disturbance is introduced into the system and the system response to the disturbance is analysed. The system response depends on the flow conditions. A signal is generated by injecting the fluid stream with a suitable tracer, the concentration of which is then measured further downstream, such as at the exit. Typical input signals include step pulse, impulse, sinusoidal and random, (Wen *et al.* [1975]).

### 4.3.2 Experimental Design and Procedures

To characterize the extent of non-ideal flow in the flotation column, the stimulus response technique was used. The general approach was to purposely excite the system by introducing a detectable substance and then observing the system response. The mixing characteristics of the system could be extracted from the response data. The stimulus used in this case was an impulse of soluble salt tracer.

#### 4.3.2.1 The Tracer

An ideal tracer should have the same flow properties as the substance or particles it represents but should not affect the transport phenomena in the equipment. The tracer should be sufficiently different in some non-flow attribute that it can be detected by an analytical instrument. It should not react with other components in the flowing material. Based on the simplifying assumption that all the components of the flowing material have the same mixing behaviour, the RTD of liquid, which is simpler to analyze,

was traced. This allowed for the possible use of water soluble electrolytes such as KCl, NaCl, and LiCl. Sodium chloride was selected for this work. A 3M NaCl solution was used.

4.3.2.2 The tests

Stimulus-response techniques were used to study the residence time distribution in the flotation column. For each test the tracer was injected at the feed entry point and the response monitored at the exit region of the column as shown in Figure 4.3. The RTD studies were done on a two phase system, water and air. The water contained some frother, equivalent to amounts used in actual flotation tests (30 micro-litres/litre). The system was run to steady state at a specified feed flow rate and 30 cc of 3 molar sodium chloride solution were injected at the feed inlet. The concentration at the exit was measured continuously. For each feed rate the experiment was performed at four different air flow rates; 0.0, 1.0, 1.5 and 2.0 litres/min. Three different feed rates were investigated; 5.5, 8.0 and 12.2 litres/min.

4.3.2.3 Conductivity Probe

A conductivity sensor was used to monitor the downstream concentration. The probe was simply an electrical circuit with a small physical gap in it as illustrated in Figure 4.4. The gap was positioned at the point of interest and a current applied to the circuit. The voltage developed

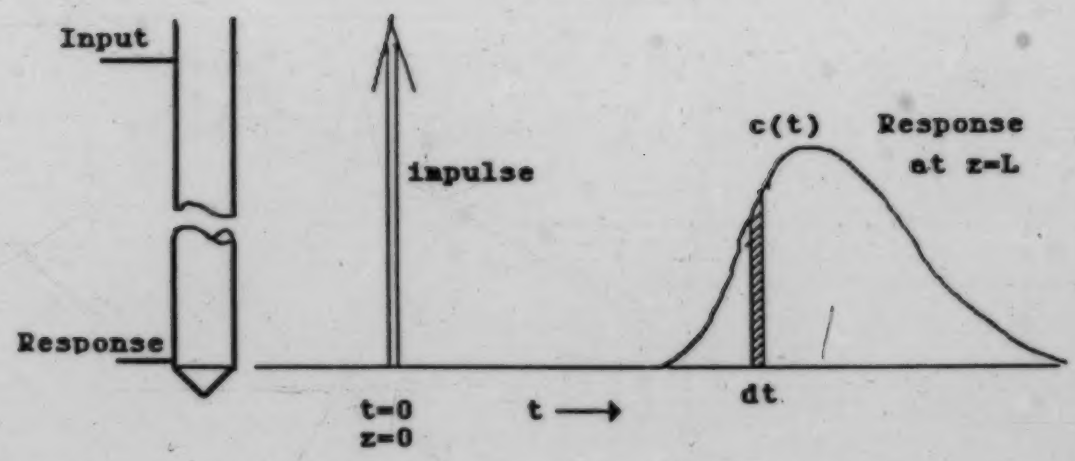


Figure 4.3 Impulse Response Method of RTD Studies

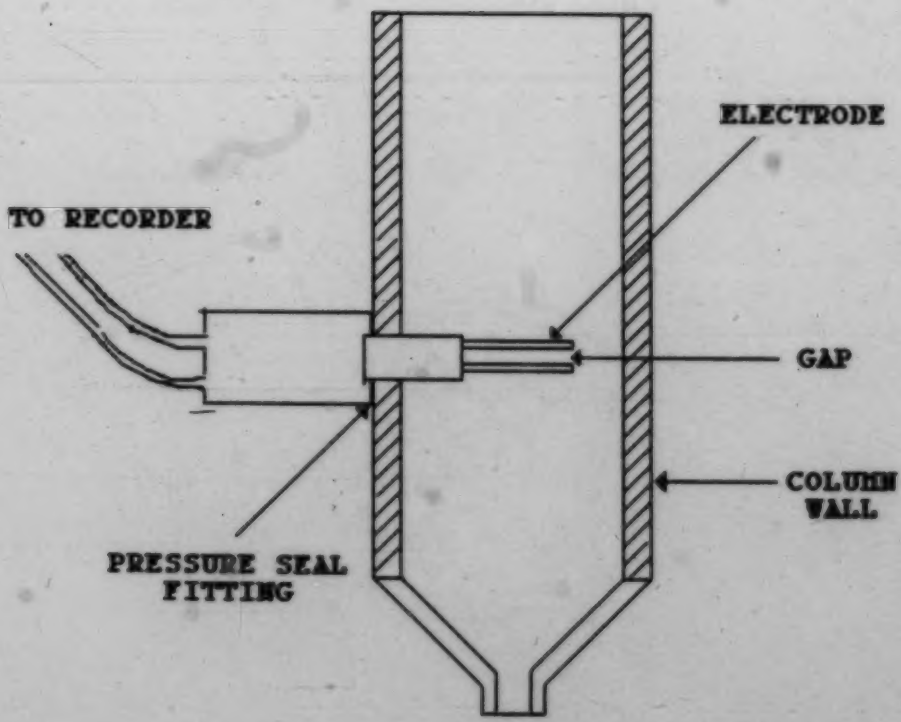


Figure 4.4 Conductivity Probe Used in RTD Tests

across the gap is a function of the electrical conductivity of the medium which bridges the gap. Since the electrical conductivity of the medium is a function of the concentration of tracer in that region, the flow dynamics could be recorded by an electrical data-acquisition system. The electrical circuit of the conductivity sensor was connected to a high speed chart recorder which continuously recorded the concentration of the tracer in the column. The resulting graphical record was digitized for further mathematical processing. Alternatively, the voltage reading across the gap could be converted to concentration in parts per million by the following equation:

$$C = \frac{10067}{V_r^{3.5}} \qquad [4.1]$$

where

V is the voltage reading

C is concentration.

This relation is valid for the conductivity probe used for this work.

### 4.3.3 Tracer Data Processing

Once the experimental impulse test data was obtained, the nominal or mean residence time $\tau$ was computed by numerical integration using Simpson's one-third rule as:

$$\tau = \frac{\int_0^\infty t\, c(t)dt}{\int_0^\infty c(t)\, dt} \qquad [4.2]$$

in which c(t) is the tracer concentration at time t. To make the data amenable for the simple integration method, the digitized data was obtained such that the intervals were equispaced. The digitized original data are plotted in Figures 4.5, 4.6, and 4.7 for the various gas flow rates.

4.3.3.1 Normalization of Data

In order to carry out statistical treatment of the response data, the experimental profile is converted into a probability density function using the following transformations:

$$E(\theta) = \frac{\tau\, c(t)}{\int_0^\infty c(t)\, dt} \qquad [4.3]$$

and:

$$\theta = \frac{t}{\tau} \qquad [4.4]$$

$E(\theta)$ is the normalized concentration and $\theta$ is the dimensionless time. The normalized data corresponding to the original data of Figures 4.5, 4.6, and 4.7 are plotted in Figures 4.8, 4.9, and 4.10 respectively.

**4.3.4 RTD Models Studied**

Several types of models can be used to characterize non-ideal flow in a vessel. Some draw an analogy between

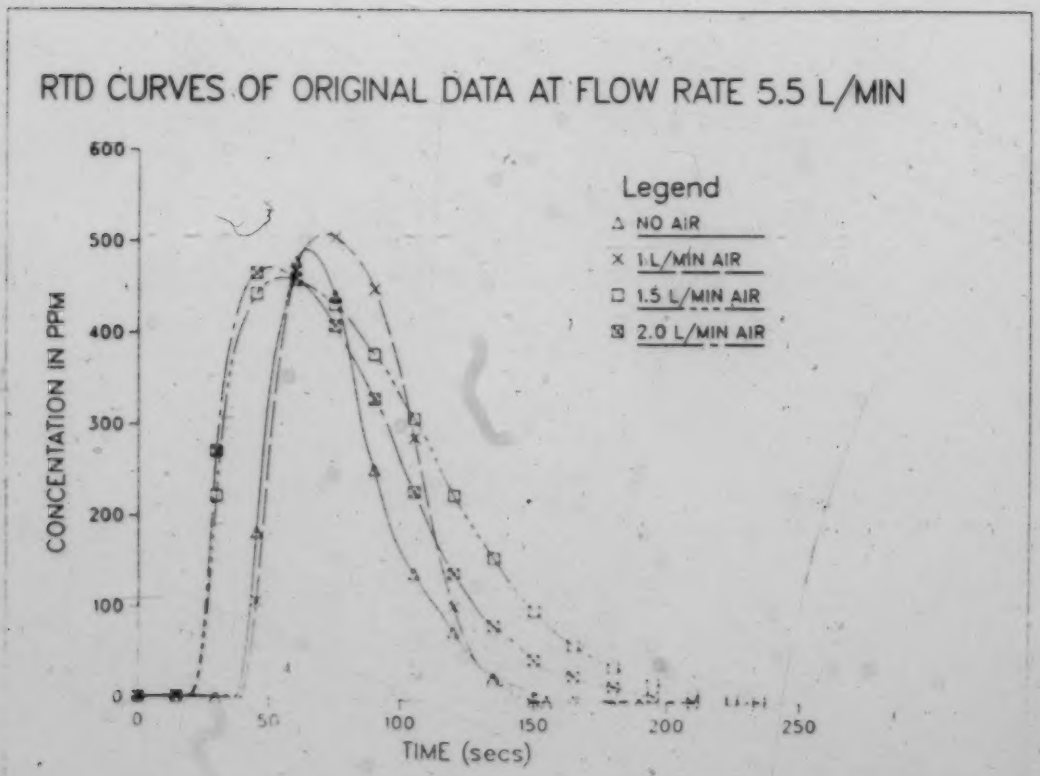


**Figure 4.5 RTD Curves of the Original Data for Various Air flow Rates and 5.5 l/min Feed Flow Rate**

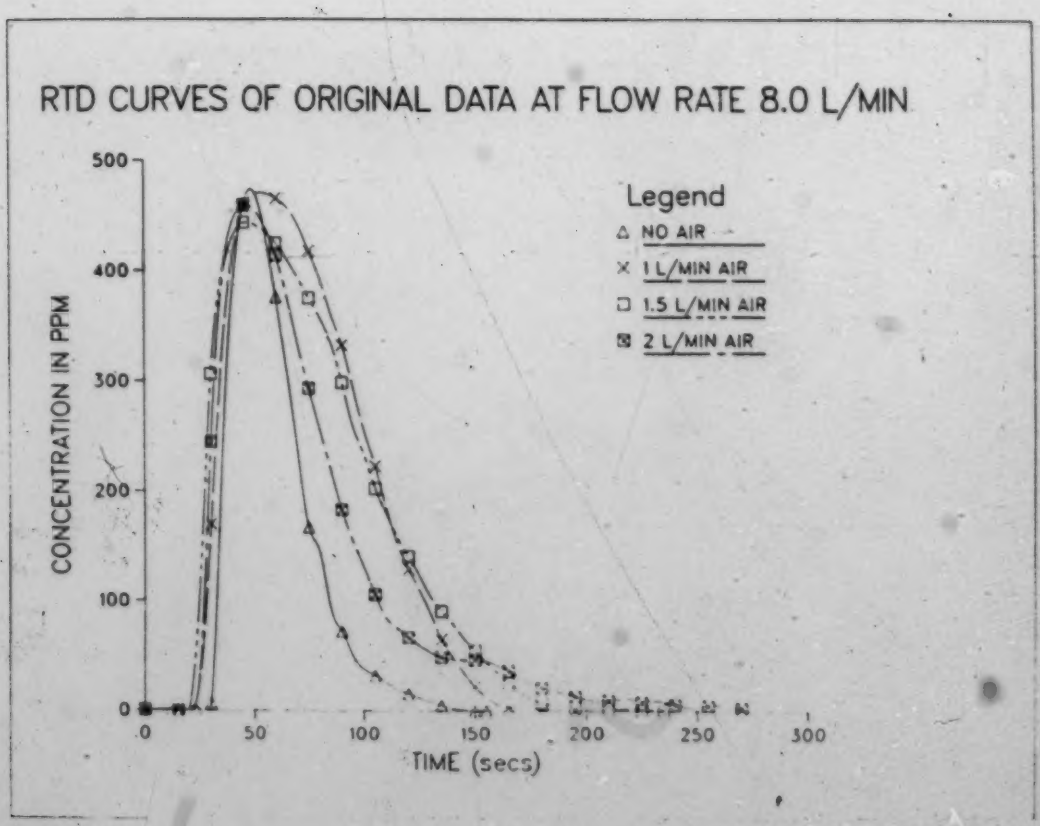


Figure 4.6 RTD Curves of the Original Data for Various Air Flow Rates and 8.0 l/min Feed Flow Rate

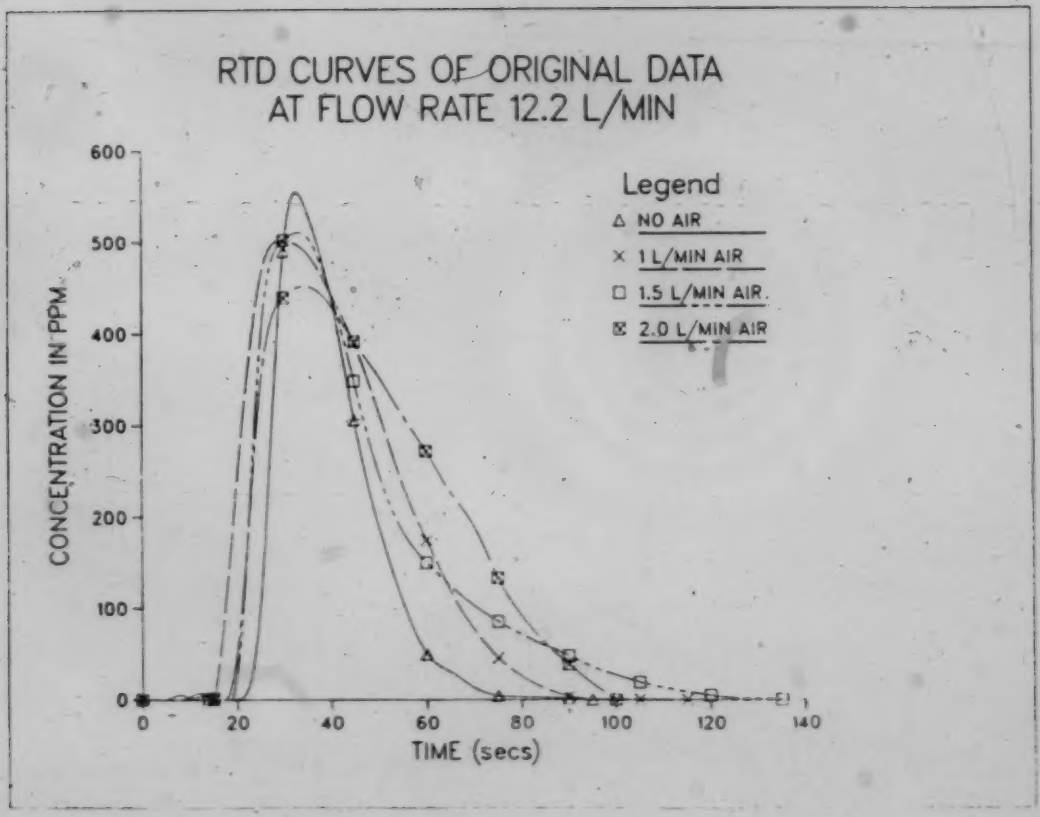


Figure 4.7 RTD Curves of the Original Data for Various Air Flow Rates and 12.2 l/min Feed Flow Rate

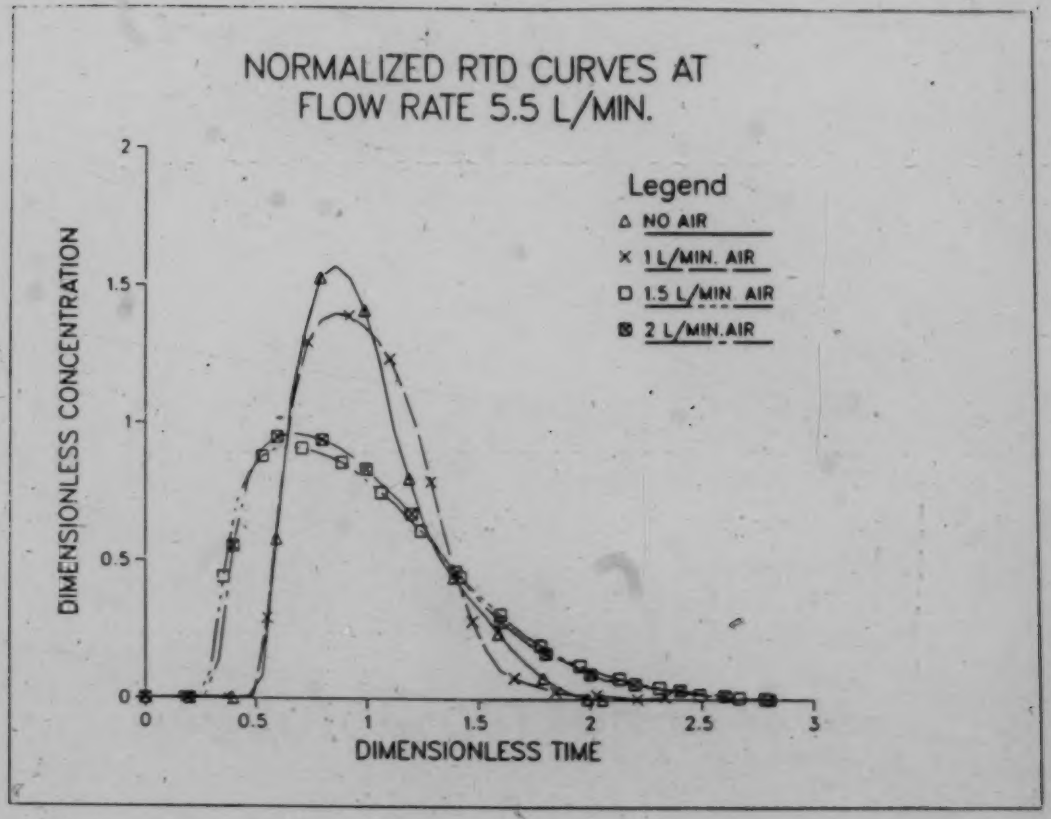


Figure 4.8 Normalized RTD Curves for various Air Flow Rates and 5.5 l/min Feed Rate

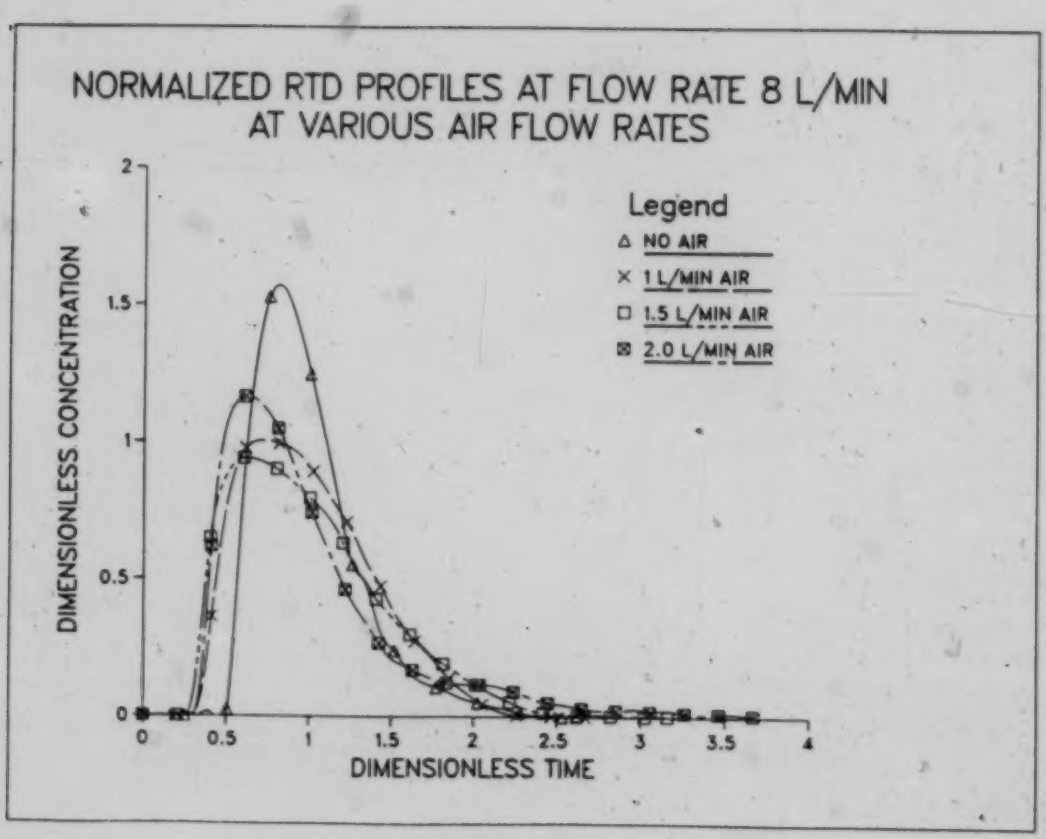


Figure 4.9 Normalized RTD Curves for Various Air Flow Rates and 8.0 l/min Feed Rate

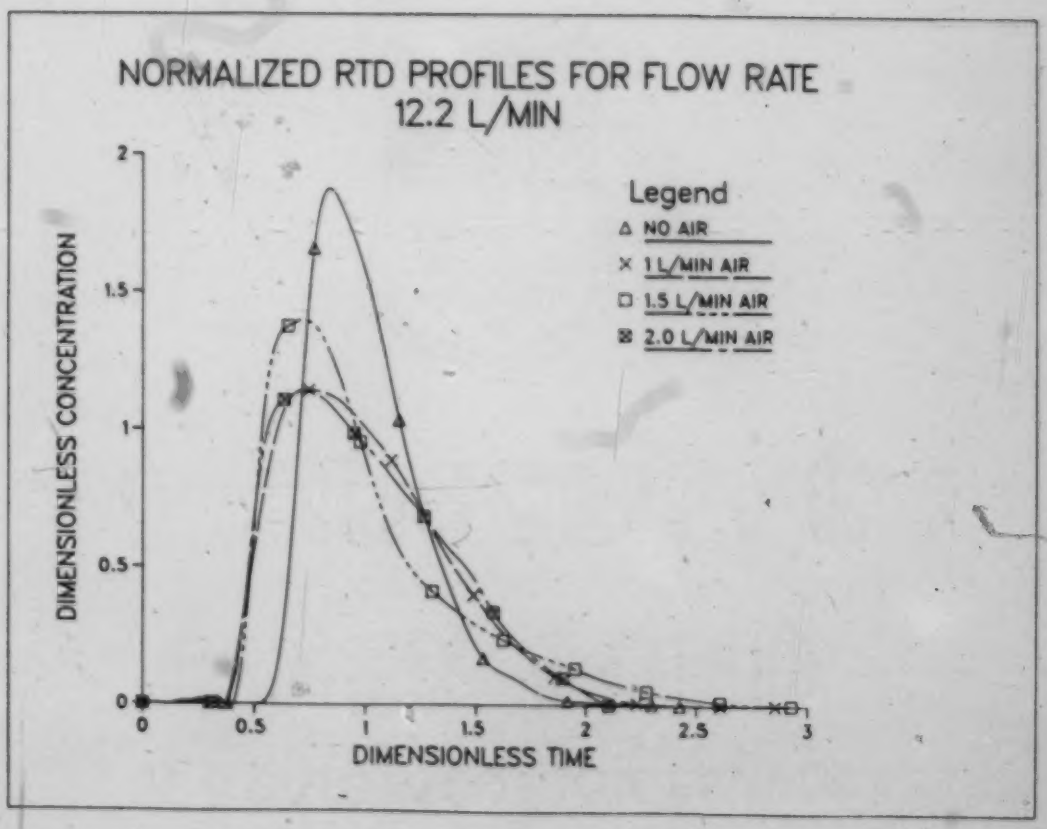


Figure 4.10 Normalized RTD Curves for Various Air Flow Rates and 12.2 l/min Feed Rate

mixing and diffusional process and these are called dispersion models. The most widely used of these types is the simple, one-dimensional, Fick's law type of diffusion equation, the axial dispersion model. Others build a chain of perfect mixers which can be equivalent (i.e, identical holdup) or not. Still others visualize various flow regimes connected in series and parallel. For the purposes of this work, the tanks in series model, the axial dispersion model and fractional tubularity model were considered due to their simplicity.

4.3.4.1 Modelling with Series of Mixers

The tanks-in-series model is a widely used one-parameter model for non-ideal flow representation. Here we view the fluid to flow through a series of equivalent ideal stirred tanks. The model can be represented in the normalized form as given in Equation 2.29. If N, the number of tanks in series is one, the result reduces to that of a single stirred tank and for large number of tanks, the tanks in series model approaches piston flow. For this model, the variance $\sigma^2$ is given by

$$\sigma^2 = \frac{1}{N} \qquad [4.5]$$

Figure 4.11 represents a comparison of the predicted RTD from all the models considered with experimental results. Figure 4.12 shows a comparison of the predicted RTD from the

tanks in series model with the observed RTD.

4.3.4.2 Fractional Tubularity Model

Another way of looking at the normalized RTD curve of a vessel is to think of it as consisting of a plug flow region and a perfectly mixed region in series. If $\tau_p$ and $\tau_m$ are the respective residence times of the plug flow and perfectly mixed sections, and $\tau=\tau_p + \tau_m$ then:

$$\theta_p = \tau_p/\tau \qquad \text{or} \quad \tau_p = \theta_p\tau$$

where $\theta_p$ is the fraction in plug flow. The plug flow section can be obtained from the normalized RTD plot. $\theta_p$ is the point on the dimensionless time axis at which the normalized concentration first becomes greater than zero. Once the plug flow section has been determined, the perfect mixing section can be analyzed. This model is usually referred to as fractional tubularity model and for the mixing section of N mixers in series, the model can be written as:

$$c(t) = 0 \qquad t \leq \tau_p$$

$$c(t) = M/(Q\tau)\,\frac{N^N((t-\tau_p)/\tau)^{N-1}\exp(-N(t-\tau_p)/\tau)}{(N-1)!} \qquad t > \tau_p \qquad [4.6]$$

where

M is mass of tracer,

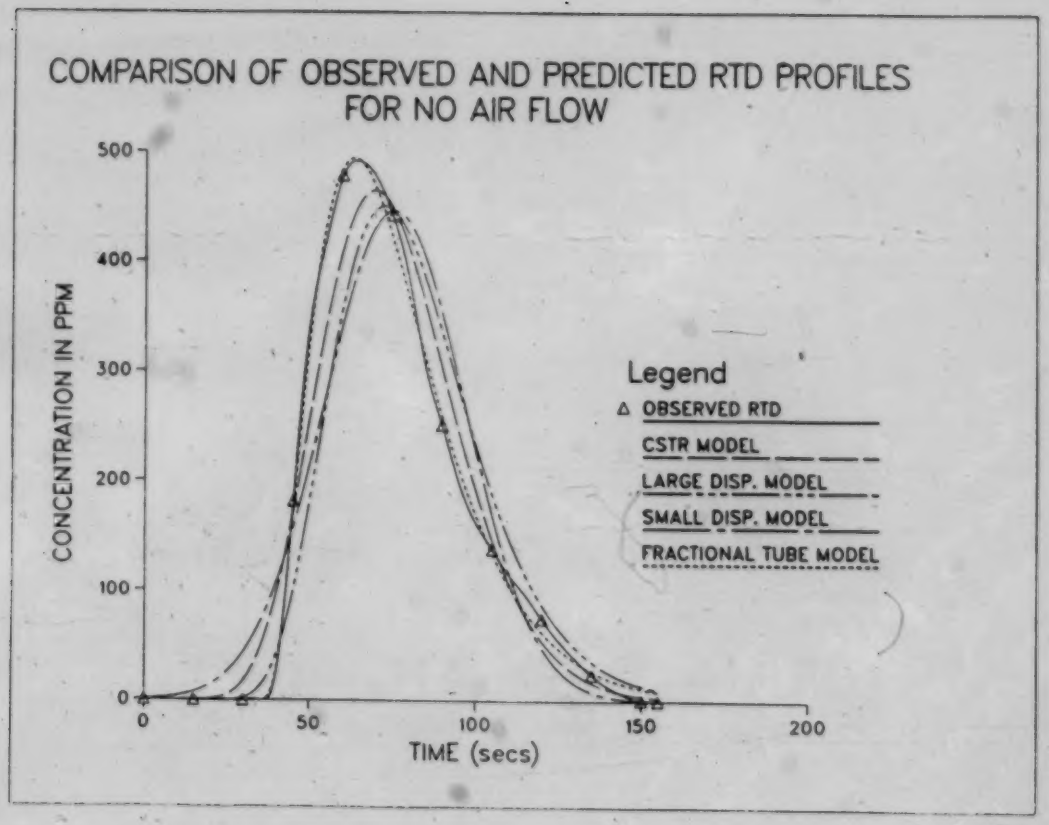


Figure 4.11 Comparison of Predicted and Observed RTD from the Various Models for a Feed Flow Rate of 5.5 l/min.

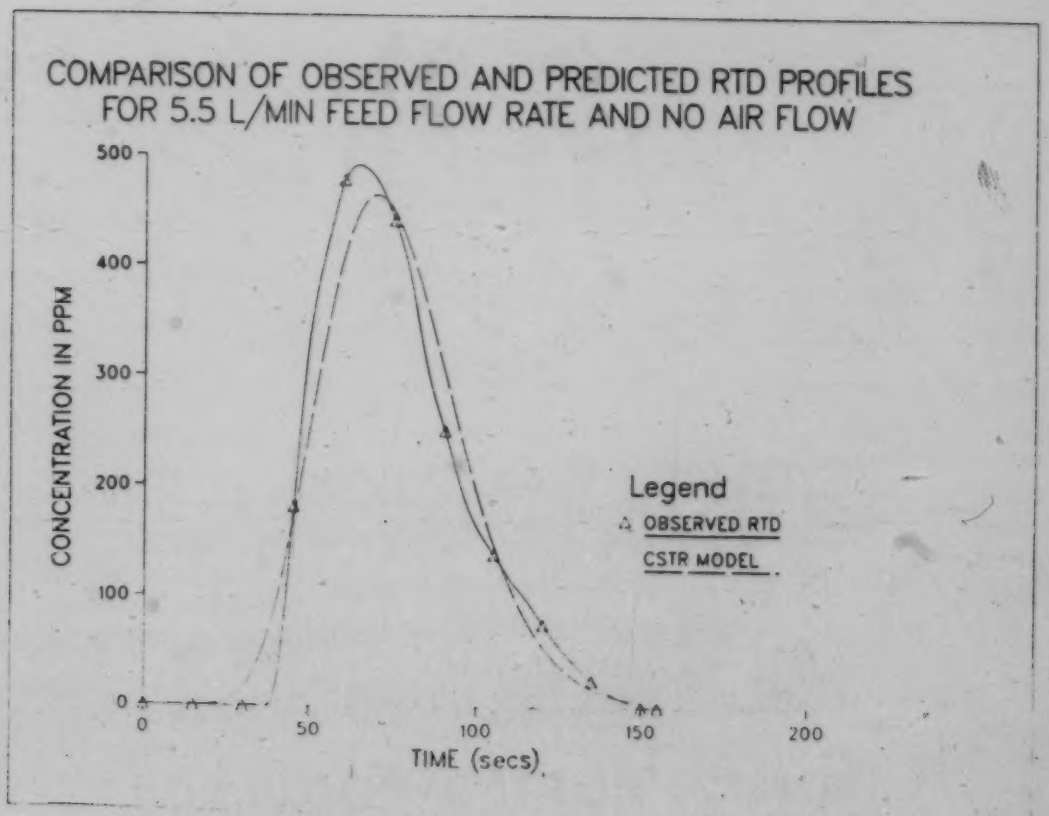


**Figure 4.12 Comparison of the Predicted RTD Profile of the Tanks in Series Model with the Observed Profile**

Q is the flow rate of material in the system.

For this model, two points can be extracted from the normalized profile; $C_{mx}$ corresponding to the maximum concentration, and $\theta_{pk}$ the time at which this concentration occurs. These points and $\theta_p$ the time corresponding to plug flow portion can be used to analyze a pure mixing section as follows:

$$\theta_m = \frac{\theta_{pk} - \theta_p}{1 - \theta_p} \qquad [4.7]$$

and;

$$N = \frac{1}{1 - \theta_m} \qquad [4.8]$$

With this approximation of N, the number of mixers, Equation 4.6 can be evaluated and estimates of the parameters, $\tau$, $\tau_p$ obtained by minimizing the objective function. The objective function here is the sums of squares of errors between the observed and predicted concentrations.

A comparison of the predicted and measured RTD curves is given in Figure 4.13. for no air and 5.5 l/min. feed rate.

4.3.4.3 Axial Dispersion Models

The axial dispersion model can be considered as the superimposition of some degree of backmixing on the plug

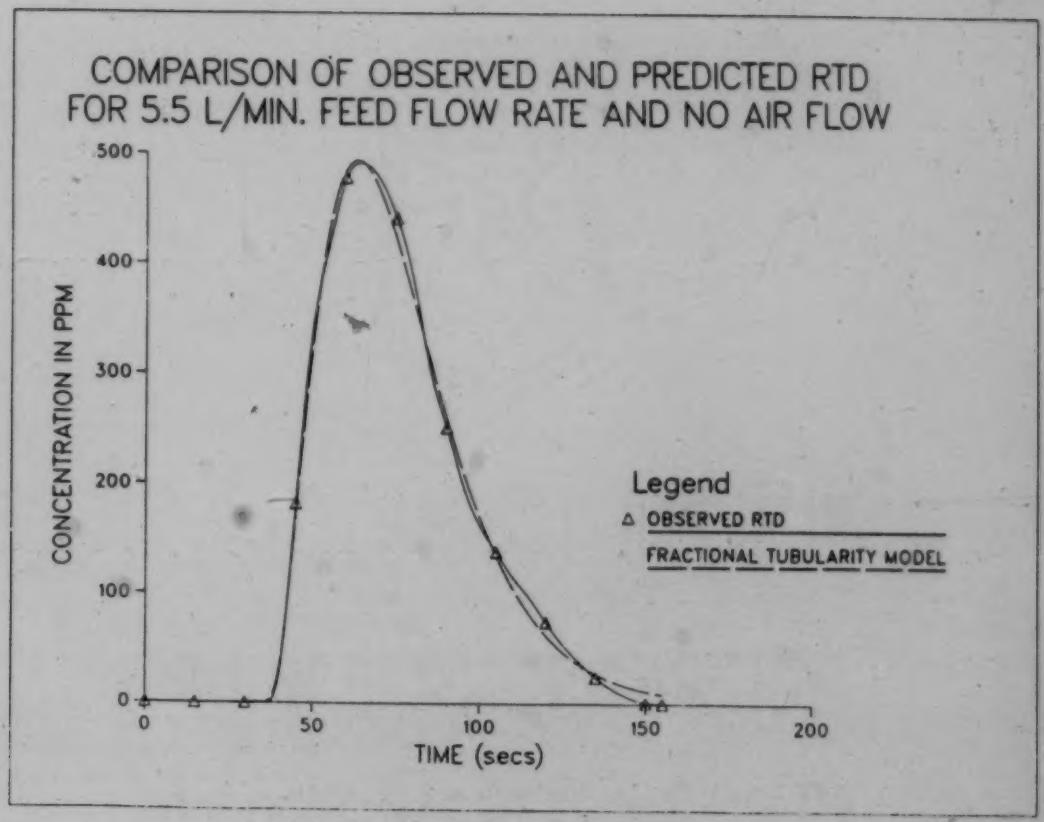


Figure 4.13 Comparison of the Predicted RTD Profile of the Fractional Tubularity Model with the Observed Profile for No Air and 5.5 l/min Feed Flow Rate

flow of fluid. This model characterizes the backmixing by a simple, one dimensional, Fick's law type partial differential equation as given in equation 2.21. The simplicity of this model has made it one of the most widely used models for correlating RTD data obtained from tracer tests. Significant inaccuracies could occur in the Peclet number obtained from the variance of the distribution if there is extensive tailing in the RTD curve, e.g, due to stagnant volumes. Since no extensive tailings was observed in the RTD profiles, the calculated Peclet numbers should be fairly accurate.

The time domain solution of the axial dispersion model for large extents of dispersion is given by (Levenspiel 1972):

$$E(\theta) = 1/2\sqrt{(\pi\theta(1/P_e))}\exp\left(\frac{-(1-\theta)^2 P_e}{4\theta}\right) \qquad [4.9]$$

where $\theta$ is the dimensionless time. For small extents of dispersion the solution of the axial dispersion model is given by the symmetrical C curve:

$$E(\theta) = 1/2\sqrt{(\pi(1/P_e))}\exp\left(\frac{-(1-\theta)^2 P_e}{4}\right) \qquad [4.10]$$

A comparison of the measured RTD and those predicted by the models for large and small extents of dispersion is given in

Figure 4.14.

### 4.3.5 Role of Models in Calculating Flotation Kinetic Parameters

Tracer information is used in conjunction with kinetic models to predict the performance of real flow reactors, including the flotation column, provided the process has first order reaction kinetics, which is the case with flotation. In other words, the tracer information can be used to scale up batch flotation results or models to continuous systems. For example, consider the batch model for recovery of a species in a flotation sytem given as:

$$R_b = R_{max}(1 - \exp(-kt)) \qquad [4.11]$$

where

$R_b$ is the recovery in the batch flotation.

To estimate the result in a continuous cell, the batch result is weighted according to the normalized RTD profile $E(\theta)$ as:

$$R_c = R_{max}\left[ 1 - \int_0^{\infty} \exp(-k(\theta\tau))\, E(\theta)\, d\theta \right] \qquad [4.12]$$

where

$\tau$ is the mean residence time,

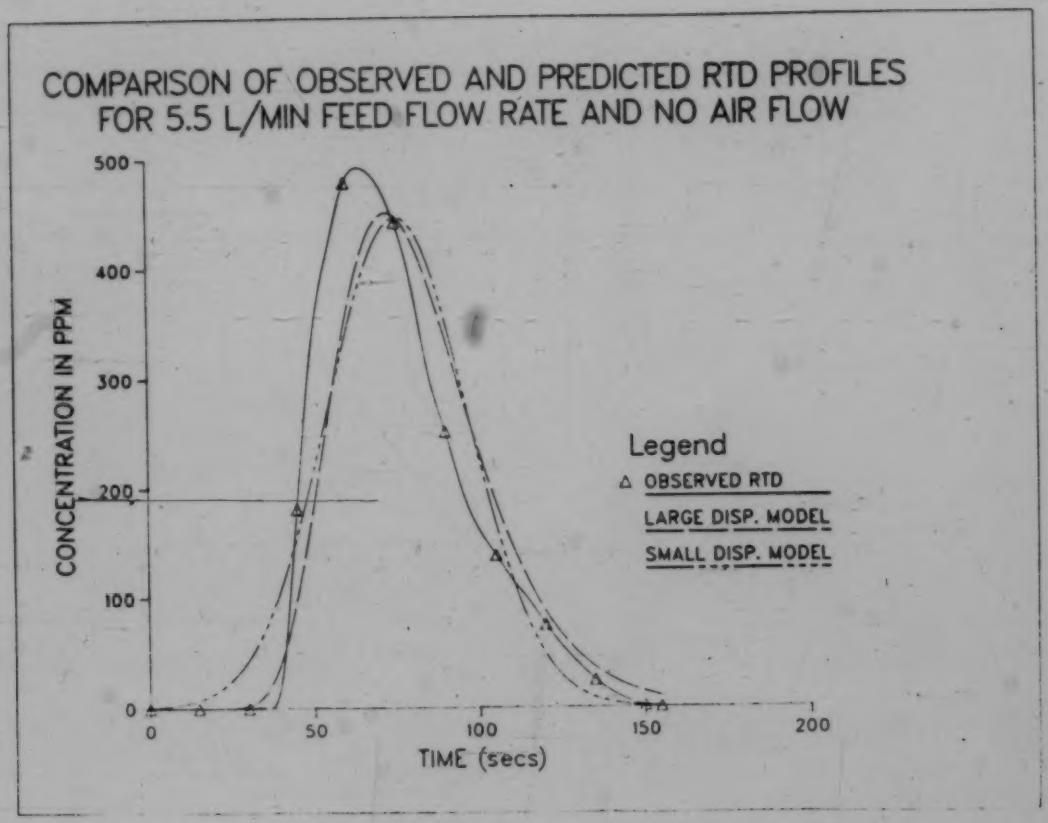


Figure 4.14 Comparison of the Predicted RTD Profile of the Axial Dispersion Models with the Observed Profile for no Air and 5.5 l/min. Feed Flow Rate

$\theta$ is the dimensionless time.

For example, if in a batch test $R_{max}$ is found to be 90%, k is 1.37 $min^{-1}$ and $\tau$ is 5 mins, the batch model predicts:

$$R_b = 89.9\%$$

If it is assumed that the continuous system profile can be described by 3 equivalent mixers in series then:

$$R_c = R_{max}\left[ 1 - \int_0^{\infty} \exp((-1.35)(5)(\tau)) \frac{3^3\theta^2\exp(-3\theta)}{(N-1)!} d\theta \right] \quad [4.13]$$

which gives $R_c = 87.46\%$ by numerical integration.

On the other hand if we assume an axial dispersion model, the recovery $R_{ca}$ is given by:

$$R_{ca} = 1 - \frac{4a \exp(P_e/2)}{(1+a)^2\exp(a\ P_e/\ 2) - (1-a)^2\exp(a\ P_e/\ 2)} \quad [4.14]$$

where a is given by equation (2.23)

An approximation of the variance from the number of cells is given by:

$$\sigma^2 = \frac{1}{N} = \frac{2}{P_e}$$

from which $P_e = 6$.

$R_{ca}$ is then 88.72%

If the fractional tubularity model is assumed, the recovery $R_f$ is given by:

$$R_f = R_{max}\left[1 - \exp(-k\tau_p)\left[\prod_{i=1}^{N} \frac{1}{(1+k\tau_i)}\right]\right] \qquad [4.15]$$

$$\sum_{i=1}^{N} \tau_i = \bar{\tau}_p = (1-\theta_p)\tau$$

If $\theta_p$ is taken to be 0.3, that is, the same as found for the column conditions shown in Figure 4.8 and 1.5 l/min air flow rate, then $\tau_p$ is 1.5 minutes. If $\theta_{pi}$ is similarily taken to be 0.7 then the perfectly mixed region can be represented by two mixers to the nearest integer from Equations 4.7 and 4.8. Then $\tau_i$ in the above equation is 1.75 minutes and the yield $R_f$ is 89.1%

The tracer information is also useful for detecting poor mixing behaviour in a reactor, such as dead or stagnant volumes that show up as long tails in the exit age distribution curve. Channelling and recirculation may also be detected by the tracer response techniques and improvement may be possible by inserting redistributors or by baffling.

### 4.3.6 Results and Discussion

The residence time distribution profiles of the original data as well as the normalized data show that there exist some dispersion or backmixing in the column. This is expected, especially with the countercurrent flow of air. It

is also seen that there are no long tails, indicating that no serious stagnant regions exist in the column at the flow rates studied. It is observed from Figure 4.15 that generally the Peclet number reduces with increasing air flow, showing that a good deal of the mixing is induced by the air. At higher air flows the Peclet number appears to increase slightly again, probably due to the onset of channelling.

The opposite effect is observed with the mean residence time as shown in Figure 4.16. As expected, the mean residence time increases with increasing air flow rate but at higher air flow rates, (greater than 1.5 l/min) it appears to decrease again. As mentioned before, this might be due to the onset of channelling causing a "fountain effect" in which the air is confined to the middle of the column while the liquid stays near the column wall, creating an impression of less mixing and shorter residence times. Since channelling is detrimental to efficient flotation, it might be necessary to operate at air flow rates at which this effect is minimal.

From the modelling point of view, Figure 14.11 shows that the residence time distribution can be fairly well described by all of the three models considered, the series of perfect mixers model, both large and small axial dispersion models and the fractional tubularity model. A chi-squared test shows that the fractional tubularity model gives the best fit. When air flows countercurrent to the

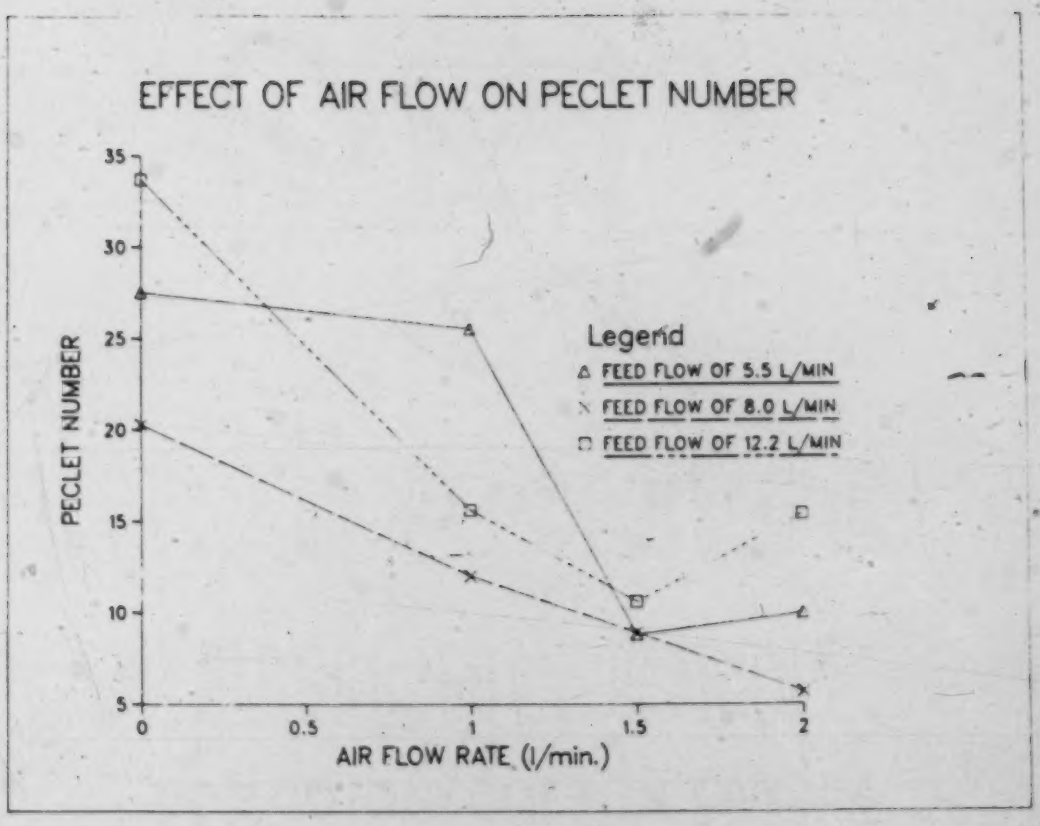


Figure 4.15 Effect of Air Flow Rate on the Peclet Number

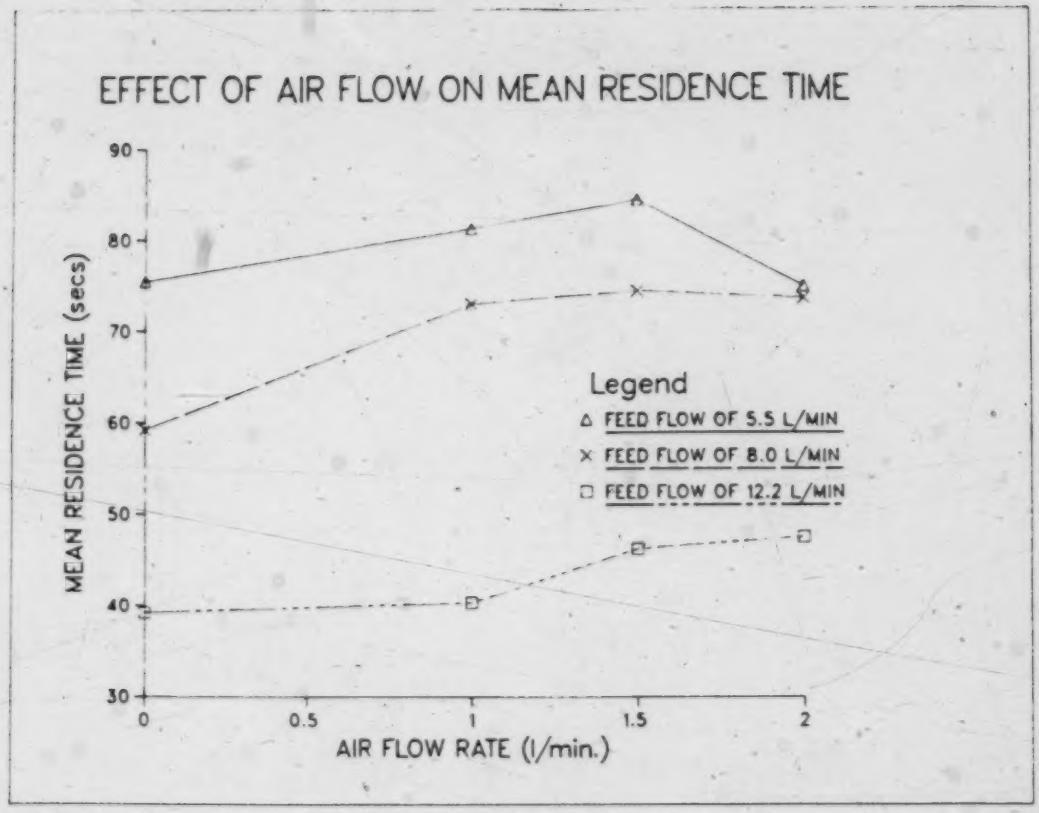


Figure 4.16 Effect of Air Flow Rate on the Mean Residence Time

liquid, the deviation of the model predictions become large and only the fractional tubularity model continues to give acceptable predictions. It can be concluded that the most appropriate RTD model for the laboratory column is the fractional tubularity model.

The study shows that though some dispersion exists in the column, the magnitude is not large enough to require baffling. The deviation of the flow from ideal plug flow may cause only minor lowering of performance. A large part of the dispersion in the column is induced by the countercurrent air flow. Air flow rates above two litres per minute are not recommended for the operations in the column used in this study because of unfavourable flow regimes.

# 5. MODELLING THE CLEANING ZONE

## 5.1 Justification of The Work

The importance of the part played by the froth phase has long been recognized (Klassen and Mokrousov [1959]), but its effect has not been explicitly included in many models that have been used to describe the flotation process. The effect of the froth phase is usually lumped together with several other effects into one parameter, such as the flotation rate constant. It is becoming increasingly clear (Cutting *et al.*[1982]) that the principal uncertainties limiting the prediction capabilities of these models are contributed by sub-processes operating in the flotation froth which has had remarkably little attention. Maksimov and Khainman (1965) showed the importance of the dropback rate from the froth to pulp by demonstrating an eighty percent return rate for the tested particles in an industrial flotation froth. They also showed that substantial increase in grade referred to as 'secondary cleaning' occurs in the froth. This has led to the awareness of the potential of the froth column as a cleaner and the need for a model which could describe this effect. With the advent of the countercurrent flotation column in which considerable froth heights are developed, the study of the processes occurring in the froth has become a necessity. Models of flotation froth provide a means of better understanding the complex interactions that lead to the

overall performance of the equipment and for scale-up or extrapolation of existing data.

## 5.2 Model Development

To evaluate a complex process such as mass transfer in flotation column froth, one must understand the factors that affect the performance of the system. One way of doing this is to employ conceptual representations of the process. An attempt is made to construct a set of mathematical relations together with boundary conditions that describe the relationships between the process variables. Because of the complexity of the process, whatever model is developed cannot be expected to completely represent the process, but should account for the important properties of the process with a reasonable degree of confidence. Usually the more complex the model the more difficult the solution. A suitable compromise must therefore be struck between the required detail and tractability and also the time available.

According to Himmelblau and Bischoff (1968), the general strategy of analysis of a process consists of the following steps:

1. Formulations of the problem and establishment of objectives.
2. Preliminary inspection and classification of the process to break it down into subsystems (elements).

3. Preliminary determination of the relationships among the subsystems.
4. Mathematical modelling of relationships in terms of variables and parameters and description of elements that cannot be adequately represented by mathematical models.
5. Evaluation of how well the model represents the process, using judgement to integrate the non-mathematical representation with the mathematical.

This is the strategy that has been used in this study.

### 5.2.1 Elements of a Conceptual Froth Model

The current understanding of the behaviour of the flotation froth from the literature and from observation can be summarized as follows:

- Air, solids and water from the recovery zone enter at the base of the froth column, and, for all purposes, have a net vertical motion upwards.
- Air reports to the concentrate stream, water and solids report either to the concentrate stream or return to the pulp.
- Air is "unusual" in that it is common to model only water and solids behavior in the recovery zone.
- One mechanism that leads to a "loss" of water and solids from the concentrate stream is bubble coalescence and collapse.
- The solids and water released due to detachment,

drainage or on bubble collapse may report to the pulp or may become part of another bubble aggregate.

- The probability of solids re-collection in the froth phase depends upon the hydrophobicity of the particles.
- Water that reports to the base of the froth column from the collection zone comes from bubble skin or in the wake of the bubbles.
- Gangue minerals concentrate in the water phase and not on the air bubbles, since they have been carried in the froth by mechanical entrainment in the water.
- Water drains from the froth phase toward the froth-pulp interface.
- The differential return rates for valuable minerals and gangue results in an increase in grade from bottom to top of froth.
- water drainage can lead to bubble film thinning and collapse.
- the addition of wash water at the top of the froth column is expected to:

  i) reduce the impact of film thinning, i.e. stabilize the froth.

  ii) minimize the "pulp water" that ultimately reports to the concentrate stream and consequently minimize gangue contamination.

### 5.2.2 Conceptual Froth Model

Based on the current knowledge of the froth zone in the flotation column summarized above, a conceptual model shown schematically in Figure 5.1 is proposed.

Assumptions made are:

1. There are three subsystems in the froth; bubbles moving upwards, liquid moving downward in the Plateau borders, and liquid moving upward in wake or films associated with the bubbles. Associated with each subsystem is a concentration of solids. (Plateau borders have been defined by Ho and Prince (1972) as the channels formed by the intersection of bubble films in a foam).
2. Longitudinal and radial dispersion can be neglected in the froth.
3. Material is transferred from the bubbles to the liquid flowing downward, and from the films which contains mostly gangue material to the downward flowing liquid. This is illustrated in Figure 5.2 for a thin slice of the froth column.
4. First order kinetics apply to the system, i.e, the mass of any species transfered is dependent on the concentration of the species.
5. A fraction of the detached solid from the bubbles re-attach themselves onto the bubbles, the probability of re-attachment depending on their degree of hydrophobicity.

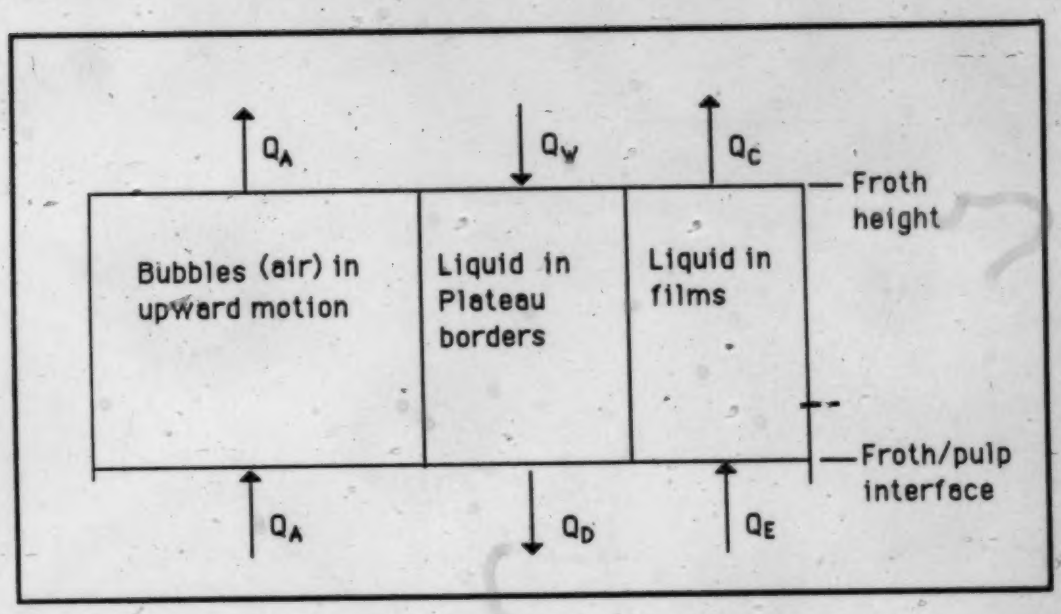


Figure 5.1 Conceptual Model of the Froth Zone

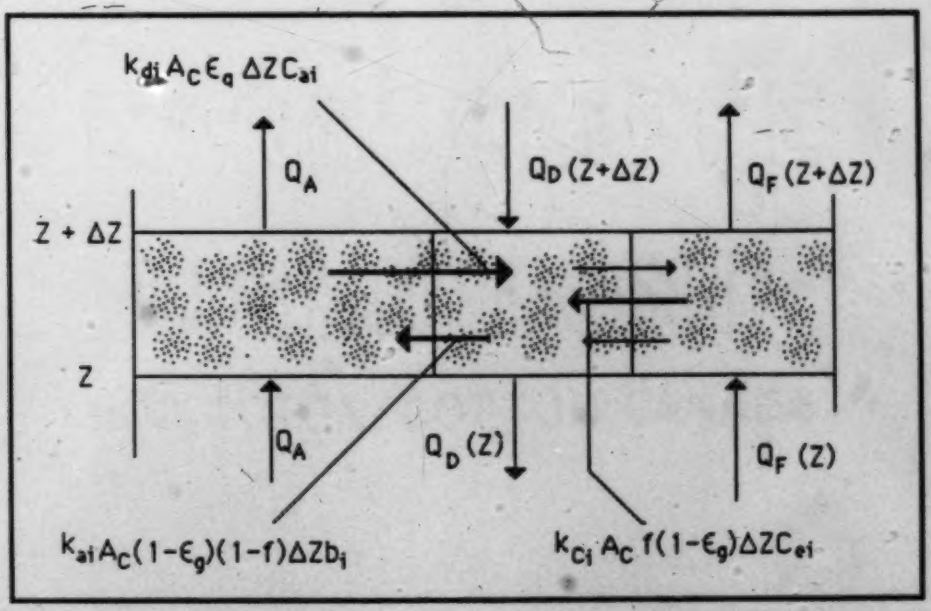


Figure 5.2 Mass Transfer in an Infinitesimal Slice of the Flotation Froth

A steady state material balance on each subsystem for a component i attached to the bubbles is given by:

$$Q_A \frac{dC_{ai}}{dz} = - k_{ai} A_c \epsilon_g C_{ai} + k_{di} A_c (1-f)(1 - \epsilon_g) C_{di} \qquad [5.1]$$

in which

$C_{ai}$ is the concentration of species i attached to air bubbles,

$C_{di}$ is the concentration of species i in the downward flowing stream,

z is the axial distance,

$k_{ai}$ is first order rate constant of detachment,

$k_{di}$ is first order rate contant of reattachment,

$A_c$ is the column cross-sectional area,

$\epsilon_g$ is the gas holdup fraction,

$Q_A$ the gas volumetric flow rate.

For component i entrained in bubble wake or in the films we have:

$$\frac{d(Q_T C_{ei})}{dz} = - k_{ci} A_c (1 - \epsilon_g) f C_{ei} \qquad [5.2]$$

where

$C_{ei}$ is the concentration of component i entrained in the wake or films of the bubbles,

$k_{ci}$ is the first order rate constant of the entrained material due to drainage,

f is the fraction of the froth liquid carried in the

bubble films,

$Q_F$ is the flow rate of the liquid carrying the entrained material in the froth.

For material in downward flowing stream due to detachment and drainage, we have, from an overall material balance across the slice of froth:

$$\frac{d(Q_D C_{di})}{dz} = Q_A \frac{dC_{ai}}{dz} + \frac{d(Q_F C_{ei})}{dz} \qquad [5.3]$$

where

$C_{di}$ is the concentration of component i in the liquid stream moving downwards in the Plateau borders.

$Q_D$ is the flow rate of the downward flowing stream.

A steady state material balance over the whole froth for species i gives:

$$Q_A C_{ai}(0)+Q_F(0)C_{ei}(0)+Q_D(Z)C_{di}(Z)=Q_A C_{ai}(Z)+Q_F(Z)C_{ei}(Z)+Q_D(0)C_{di}(0) \qquad [5.4]$$

where

$\overline{Z}$ represents the total froth height.

5.2.2.1 Boundary Conditions

The initial concentration of floatable species i entering the froth, $C_{ai}(0)$, as well those of entrained species i $C_{ei}(0)$ must be known in order to obtain solutions to the model. It was assumed, following Moys (1978) that

mineral particles attach to the bubbles in the pulp phase at a rate proportional to the available surface area of bubble, $A_b$, in the column and concentrations $C_{fi}$ of particles in the feed to the column, and rise at a rate, $Q_A C_{ai}(0)$ into the froth.

Then:

$$Q_A C_{ai}(0) = k_i A_b C_{fi} \qquad [5.5]$$

where $A_b$, the bubble surface area is given by:

$$A_b = \frac{6\tau Q_A}{d} \qquad [5.6]$$

Then:

$$C_{ai}(0) = \frac{6(k_i \tau) C_{fi}}{d} \qquad [5.7]$$

where

$k_i$ is the rate constant for species i

$\tau$ is bubble residence time in the pulp

d is the mean bubble diameter.

For entrained gangue and other minerals, the initial flow

rate is given by:

$$Q_E = \frac{6Q_A\delta}{d} \qquad [5.8]$$

so that:

$$C_{ei}(0) = \frac{(k_i\tau)C_{fi}}{\delta} \qquad [5.9]$$

It is also assumed that all gangue materials enter the froth by entrainment.

The concentration of a species in the downward flowing stream at the pulp/froth interface $C_{di}(0)$ can be estimated from the steady state overall balance and knowledge of solids concentration in the concentrate.

Transfer of entrained material

The rate coefficient $k_{ei}$ can be related to the downward liquid flow rate by arguing that the entrained particles experience a shear force due to the liquid motion. Bascur(1982) has used the relationship:

$$k_{ei} = K_{ei} d_p \rho V_D \qquad [5.10]$$

where

$K_{ei}$ is a rate constant,

$d_p$ is particle diameter,

$\rho$ is particle density,

$V_D$ is the downward bulk fluid flow.

## 5.3 Hydraulic Model

The steady state overall water balance in the froth is given by:

$$Q_E + Q_W = Q_D + Q_C \qquad [5.11]$$

where

$Q_E$ is the flow rate of liquid into the bottom of the froth layer in the bubble films,

$Q_W$ is wash water flow rate into the top of the froth,

$Q_D$ is flow of water in drainage in the Plateau borders,

$Q_C$ is the concentrate water flow rate.

The amount of water in the films entering the froth has been calculated as (Bascur[1982],Moys[1978] ):

$$Q_T(0) = \frac{6Q_A\delta}{d} \qquad [5.12]$$

in which

$Q_A$ is the volumetric air flow rate,

$Q_E$ is the same as $Q_T(0)$, the flow rate of pulp into the froth by entrainment.

d is the mean size bubble diameter in the collection zone

$\delta$ is the film thickness.

The equation is obtained from the number of bubbles rising per unit time, $6\ Q_A/\pi d^3$ and the volume of liquid held by a film surrounding a spherical bubble, $6\times0.288d^2\delta$.

$Q_C$, which is measurable, can be estimated as a first approximation from a modified form of the equation of a rectangular weir:

$$Q_C = k_h \pi D\ T^{1.5}(1 - \epsilon_g) \qquad [5.13]$$

where

$k_h$ is an adjustable constant,

D is the cell diameter,

T is the head of froth above the top of the column.

Steiner *et al.*(1977) have expressed the flow rate of the downward flowing stream $Q_D$ as:

$$Q_D = \frac{8.845\times10^{-3}\epsilon_g^2\rho g\ r^4 A_C}{\mu k_v d^2} \qquad [5.14]$$

where

$\rho$ is the liquid density,

g is the gravitational acceleration

r is the radius of Plateau border,

$\mu$ is the fluid viscosity,

$k_v$ is an an adjustable velocity parameter.

Rearranging equation 5.11, substituting for $Q_F$ and $Q_D$ from equations 5.12 and 5.14 and solving for r, the radius of Plateau border, yields:

$$r = \left[ 113.1 \frac{\mu d^2 k_v}{\rho g \epsilon_g^2 A_c} \left[ 3.3 Q_A \delta/d - (Q_c - Q_v) \right] \right]^{0.25} \qquad [5.15]$$

The total liquid holdup in the froth has been given by Hartland and Barber as:

$$(1 - \epsilon_g) = 3.3 \epsilon_g \frac{\delta}{d} + 1.26 \epsilon_g \frac{r^2}{d^2} \qquad [5.16]$$

Substituting for r from equation 5.15 yields:

$$(1 - \epsilon_g) = 3.3 \epsilon_g \frac{\delta}{d} + \frac{1.26}{d^2} \left[ 113.1 \frac{\mu d^2 k_v}{\rho g A_c} \left[ 3.3 Q_A \delta/d - (Q_c - Q_v) \right] \right]^{0.5} \qquad [5.17]$$

Film Thinning and Coalescence

One mechanism that leads to mass transfer from the bubbles and associated liquid moving upwards to the liquid moving downwards is bubble film thinning leading to coalescence or breakage.

The phenomenon of film thinning resulting in coalescence or breakage has been described by various researchers in the literature (Hartland & Barber [1974]; Steiner *et al.* [1977] Bikerman J.J.[1973]).

Following Hartland and Barber (1974), the rate of film thinning is described by the Reynolds equation:

$$\frac{d\delta}{dt} = -\frac{29.14\delta^3\sigma}{nf^2\mu d^2 r} \qquad [5.18]$$

where $\sigma$ is the surface tension of the fluid,

nf is an adjustable parameter.

Also:

$$\frac{d\delta}{dz} = \frac{d\delta}{dt} \cdot \frac{dt}{dz} \qquad [5.19]$$

At steady state, the upward velocity of the films is equivalent to the bubble velocity:

$$\frac{dz}{dt} = \frac{Q_A}{A_c \epsilon_g} \qquad [5.20]$$

Substituting equations 5.18 and 5.20 into 5.19 gives:

$$\frac{d\delta}{dz} = -\frac{29.14\delta^3\sigma A_c \epsilon_g}{nf^2\mu d^2 Q_A r} \qquad [5.21]$$

Fraction of froth liquid associated with bubble films

From the conceptual representation of the froth zone, water and air flows for an infinitesimal slice would appear

as shown in Figure 5.3.

The gas flow rate in the froth $Q_A$ is assumed constant, independent of height. It is noted that, because of their association, the film water velocity must equal the air velocity, so:

$$\frac{Q_A}{A_c \epsilon_g} = \frac{Q_F}{A_c (1 - \epsilon_g)\ f} \qquad [5.22]$$

where

$A_c$ is the column cross-sectional area,

f is the fraction of the water phase occurring as bubble film water.

Then:

$$Q_F = Q_A \left(\frac{(1 - \epsilon_g)}{\epsilon_g}\right) f \qquad [5.23]$$

Since $Q_F$ is equal to $Q_E$ at z=0, combining with Equation 5.12 gives:

$$f = \left\{\frac{6\ \delta}{d}\right\}\left\{\frac{\epsilon_g}{(1 - \epsilon_g)}\right\} \qquad [5.24]$$

$$0 < f < 1$$

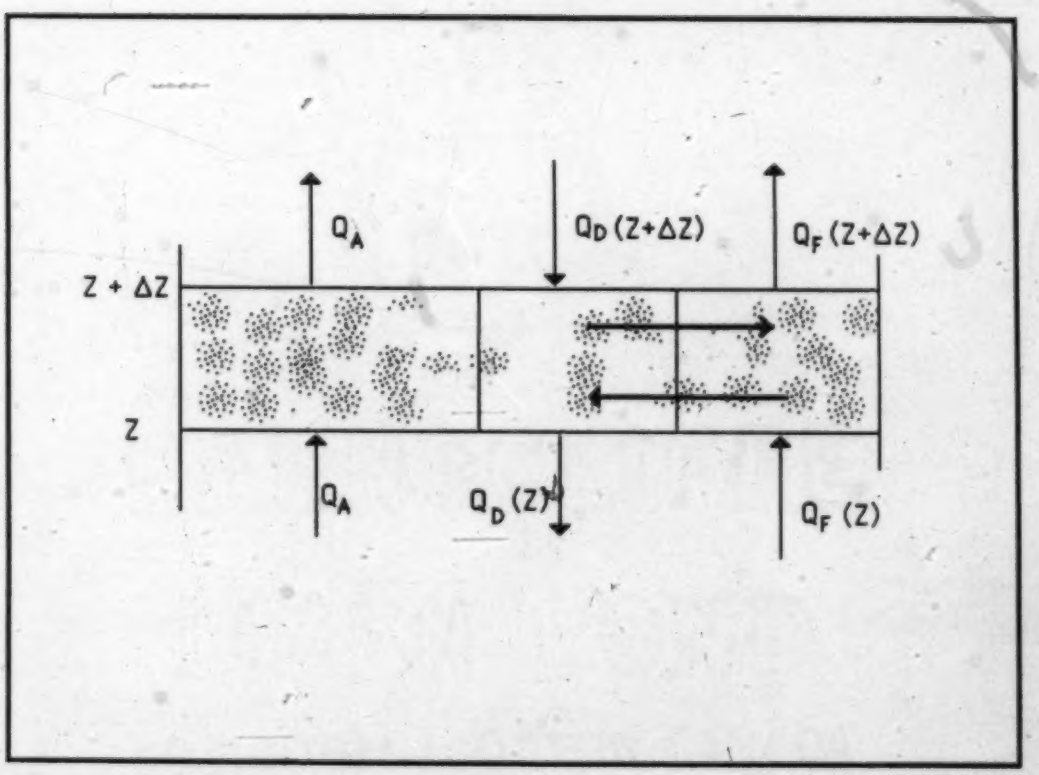


Figure 5.3 Interstitial-Film Water Transfer in an Infinitesimal Slice of the Flotation Froth

## 5.4 Solution of Model

Given the physical parameters $Q_A$, $Q_c$, $Q_w$, $A_c$, d, $\rho$, $\sigma$, $\mu$, g, nf, $k_v$, and initial estimates of $\delta$, and $\epsilon_g$ at the pulp froth interface, an estimate of r can be obtained from equation 5.16. With these values, equation 5.21 can be numerically integrated for $\delta$, equation 5.18 can then be solved for $\epsilon_g$ and equation 5.15 can be solved for r. At each step in the integration process, the water flows are available from equations 5.12 and 5.14. With the values of $\epsilon_g$ and $\delta$, f, the fraction of water in the froth which is carried in the films can be estimated from Equation 5.24.

The solids phase equations can then be solved.

Solids

With the initial flow rates of the upward and downward flowing liquids known, and knowing the initial solids concentrations from Equations 5.7 and 5.9 and the overall material balance equation 5.4, the model solution can be obtained.

The solids concentration profile can be calculated from the mass balance Equations 5.1, 5.2, and 5.3. At each stage, the fractional air holdup and the fraction of liquid in the films are calculated. The computation of the solids concentration is done for each species present and the total concentration for species i is found from

$$C_i = \epsilon_g C_{ai} + f(1-\epsilon_g)C_{ei} + (1-\epsilon_g)(1-f)C_{di} \quad i=1,2,\ldots,j \qquad [5.25]$$

where j is the number of species present with $C_{ai}=0$ for the gangue by assumption.

The mass is found by multiplying the concentrations by the elemental volumes of the subsystems. The mass fraction of a species is then found by dividing the mass of that species by the total mass in that volume element.

## 5.5 Experimental Design and Procedure

### 5.5.1 Equipment:

The flotation column described in Chapter 3 was used in these tests with the following important modifications:

a) A vacuum arrangement for taking representative samples was designed and installed on the flotation column. A schematic diagram of the arrangement is shown in Figure 5.4. This arrangement is similar in design to that used by Cutting *et al.* (1982).

b) Two manometers were installed on the column, one at the bottom and one at the top. The difference in levels was used to compute an estimate of gas holdup fraction in the recovery zone of the column.

### 5.5.2 Sample Preparation

The bulk sample of about two hundred and twenty six kilograms (500 lbs) coal was received from the Crows Nest Resources' Line Creek coal preparation plant. It had been taken in small increments from the plant's flotation concentrate. The characteristics of the bulk sample are summarized in Table 5.1 based on analysis of a head sample taken from this bulk sample. The bulk sample was air dried, split to the quarter by cone and quartering and then riffled down to a sixteenth. These sub-samples were then bagged and stored.

Table 5.1 Bulk Feed Sample Characteristics

| SIZE TYLER MESH | FRACTIONS MICRONS | MASS % | ASH% | CUM. % PASSING |
|---|---|---|---|---|
| +65 | +210 | 28.45 | 7.4 | 71.55 |
| –65+100 | -210 +149 | 16.66 | 9.16 | 54.89 |
| –100 +150 | -149 +105 | 15.46 | 9.76 | 39.43 |
| -150 +200 | -105 +74 | 14.63 | 11.09 | 24.8 |
| -200 +325 | -74 +44 | 9.07 | 13.8 | 15.73 |
| -325 | -44 | 15.73 | 14.5 | -- |
| OVERALL | | 100 | 10.29 | |

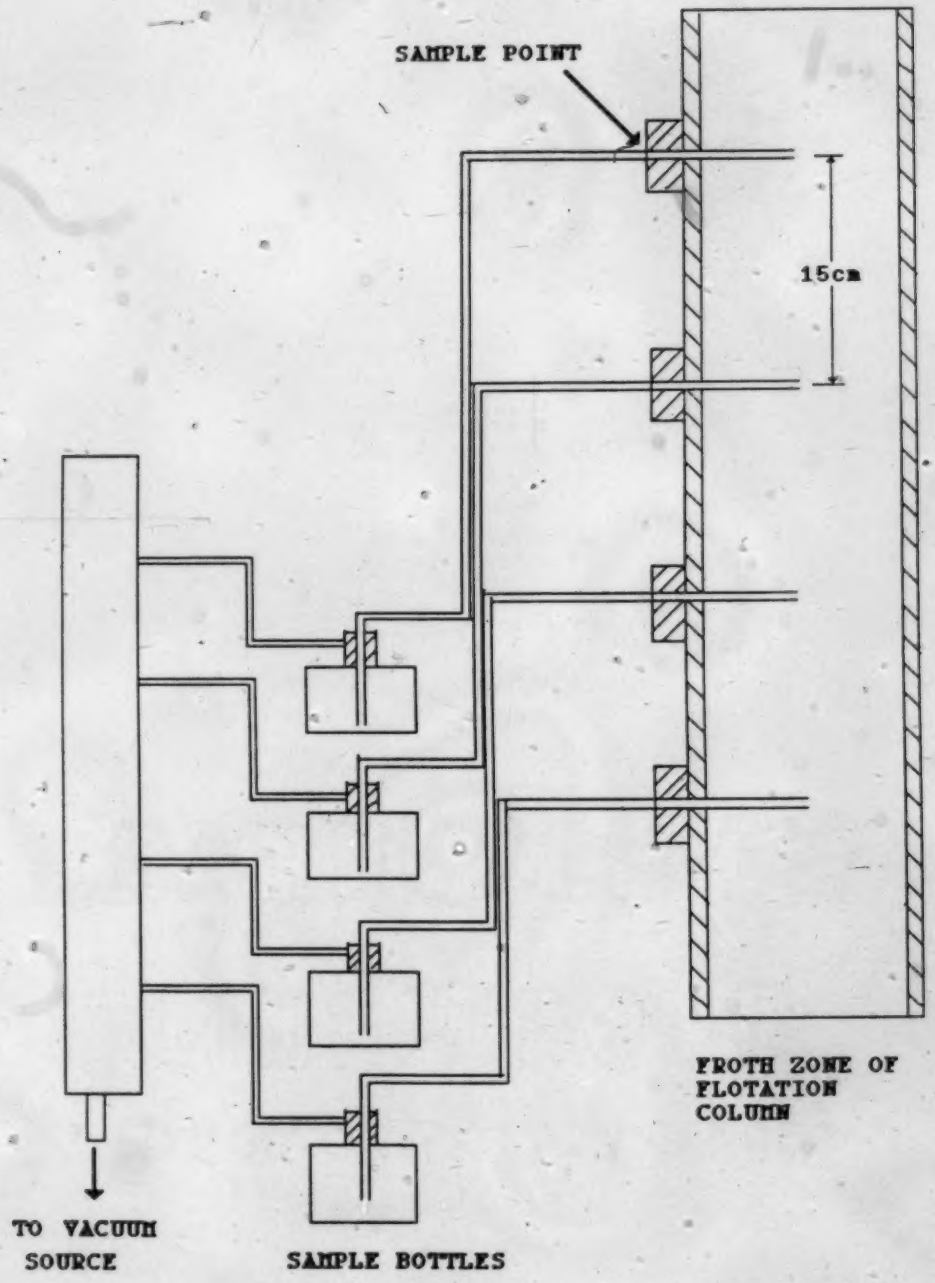


Figure 5.4 Schematic Diagram of the Vacuum Sampling Arrangement

Feed for the flotation experiments was prepared by the homogenization and representative subsampling of coal from the bags. A model fine coal was prepared from this by adding fine silica to bring the total ash to approximately twenty percent. The silica was made up of forty percent -400 mesh, 30% of -200 + 400 mesh and 30% -70 + 140 mesh. This artificial mixture was preferred to a run of mine coal because it was judged to be especially useful in documenting the washing capability of the flotation column. In the flotation tests, only the operational factors were investigated because the chemical factor effects are the same in conventional cells as in columns. The factors investigiated were the feed flow rate, wash water addition rate and air flow rate. An orthogonal factorial design was used with two levels for feed and air flow rate and three levels for wash water addition rate as shown in Table 5.2.

### 5.5.3 Test Procedures

The flotation column experiments were conducted by first filling the 370 litre feed conditioning tank with about eighty eight litres of water. The appropriate quantity of feed solids was then added to give the required solids concentration (5% by weight), while the mixer was running. The required amount of kerosene collector was added and then the frother (4-methyl 2-pentanol) (MIBC). The column itself was then filled with water from the wash water line and then the wash water was adjusted to the desired flow rate. The

Table 5.2 Experimental Design for Runs 1 to 18

FIRST SERIES

| RUN | FEED RATE (l/min) | AIR FLOW (l/min) | WASH WATER FLOW(L/min) | YIELD % | ASH % |
|---|---|---|---|---|---|
| 1 | 2.8 | 1.5 | 0 | 86.79 | 9.18 |
| 2 | 3.5 | 1.5 | 0 | 86.55 | 8.83 |
| 3 | 2.8 | 2 | 0 | 86.56 | 8.7 |
| 4 | 3.5 | 2 | 0 | 86.6 | 9.06 |
| 5 | 2.8 | 1.5 | 0.75 | 85.69 | 8.2 |
| 6 | 3.5 | 1.5 | 0.75 | 85.3 | 7.67 |
| 7 | 2.8 | 2 | 0.75 | 85.89 | 8.23 |
| 8 | 3.5 | 2 | 0.75 | 85.85 | 8.41 |
| 9 | 2.8 | 1.5 | 1.5 | 86.03 | 8.2 |
| 10 | 3.5 | 1.5 | 1.5 | 86.9 | 8.99 |
| 11 | 2.8 | 2 | 1.5 | 84.66 | 8.1 |
| 12 | 3.5 | 2 | 1.5 | 86.03 | 9 |
| 13 | 3.5 | 1.5 | 0 | 87 | 9 |
| 14 | 2.8 | 2 | 1.5 | 85.34 | 8.57 |
| SECOND SERIES | | | | | |
| 15 | 4.5 | 2 | 1.5 | 72.68 | 7.73 |
| 16 | 3.5 | 2 | 0.75 | 75.13 | 8.18 |
| 17 | 4.5 | 2 | 0 | 76.37 | 9.41 |
| 18 | 4.5 | 2 | 0.75 | 73.22 | 9 |

air flow rate was also set at the desired level. After the feed slurry was conditioned for at least ten minutes, the feed pump was turned on and the flow adjusted to the desired operating level. The interface level in the column was set by controlling the speed of the tailings pump. Sampling was commenced at least five minutes after the first froth overflows into the overflow lauder. A preliminary test to determine the time to steady state showed that the system took three to four minutes to do so. In this test, samples were taken from the clean coal product at one-minute intervals for twenty minutes. The samples were ash analyzed and the time after which the ash content of the product stayed practically constant was taken to be the steady state time for the system.

5.5.3.1 Sampling

Samples were removed through sampling ports located at 15cm intervals on the column. Four samples are taken from the froth or cleaning zone as well as the concentrate sample. One sample was taken from the collection zone just below the interface and two tailings samples were taken; at the bottom of the column and from the tailings line. The samples were weighed, filtered and dried and the solids concentrations profile in the froth zone determined. Ash analysis of the samples was done in duplicates to determine the ash content and thence calculate the yield. The yields were computed from the ash analysis results because the tailings flow rate was manipulated to control the level of

the pulp/froth interface and so computation of yield from the slurry or solid flows was considered inaccurate. The percent solids in the tailings was too small to give a good estimate of the yield.

### 5.5.4 Results and Discussions

The column flotation results for the fourteen runs are given in Appendix B. The results of the additional four runs with a higher feed ash content are presented in Tables 5.3, 5.4, 5.5, and 5.6. The feed ash content of the first set of fourteen tests were found to be too low and the results could not be used for the model verification. Consequently the second set of four tests was run for that purpose. The operating conditions which were varied in the first set of tests were wash water, air and feed flow rates. Two levels each of air and feed flow rates and three levels of wash water flow rates were used.

General Observations

As previously mentioned,the feed material used in this study was a "model coal" prepared by mixing the coal sample from the Line Creek preparation plant clean coal with a pre-determined proportion of silica to give the specified feed ash. It was assumed that the reagent dosages used by the Line Creek preparation plant (3.7 cc kerosene/kg solids and .22 cc MIBC/kg solids) could be appropriately applied to these tests. However, it was observed that due to the high

Table 5.3 Column Flotation Results for Run 15

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 4.5 L/Min | Feed Ash: | 30% |
| Wash Water Rate: | 1.5 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 16.01 | 9.42 |
| 2 | 10.18 | 15.71 |
| 3 | 10.35 | 16.71 |
| 4 | 8.46 | 19.03 |
| 5 | 8.35 | 16.91 |
| 6 | 7.73 | 30.42 |
| 7 | 89.24 | 0.78 |

Product Yield = 72.68%

Table 5.4 Column Flotation Results for Run 16

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 3.5 L/Min | Feed Ash: | 30% |
| Wash Water Rate: | 0.75 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 18.95 | 12.30 |
| 2 | 11.78 | 24.94 |
| 3 | 10.02 | 22.43 |
| 4 | 9.01 | 23.61 |
| 5 | 8.88 | 24.17 |
| 6 | 8.18 | 29.55 |
| 7 | 95.92 | 0.80 |

Product Yield = 75.13%

Table 5.5 Column Flotation Results for Run 17

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 4.5 L/Min | Feed Ash: | 30% |
| Wash Water Rate: | 0.0 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 17.95 | 12.15 |
| 2 | 12.04 | 30.33 |
| 3 | 11.69 | 28.14 |
| 4 | 10.97 | 31.47 |
| 5 | 10.27 | 32.72 |
| 6 | 9.41 | 36.00 |
| 7 | 96.53 | 0.98 |

Product Yield = 76.37%

Table 5.6 Column Flotation Results for Run 18

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 4.5 L/Min | Feed Ash: | 30% |
| Wash Water Rate: | 0.75 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 19.90 | 9.87 |
| 2 | 12.61 | 19.76 |
| 3 | 11.51 | 20.64 |
| 4 | 10.43 | 21.85 |
| 5 | 9.54 | 18.93 |
| 6 | 9.08 | 33.03 |
| 7 | 87.40 | 0.89 |

Product Yield = 73.22%

floatability of the coal and the relatively small percentage ash, froth overloading occurred. The bubbles were so heavily loaded with coal that 'freezing' of the froth zone resulted, reducing the froth mobility quite drastically. The problem was alleviated only after the frother dosage had been increased nearly three times and the feed percent solids reduced from 7.5 to 5.0.

It is noteworthy to observe from Table 5.2 that the product ash obtained for all the experiments were well below the 10.29% contained in the Line Creek plant product. This is probably an indication that the ash of 10.29% could be significantly reduced by column flotation.

Effect of feed flow rate

The effect of feed flow rate is not immediately apparent but a stepwise multiple linear regression of yield on the above operating conditions yielded the following correlation:

$$\text{Yield} = 86.595 - (0.3749) \times \text{wash water rate} \times \text{air rate} \qquad [5.26]$$

It appears therefore that the slurry feed rate does not have much influence on the product yield. This implies that one may want to operate at the high flow range as long as adequate residence time is maintained. This may be true for feed material with slow flotation kinetics. For highly

floatable material such as the coal used in these tests, there is a risk of froth overload and must be considered in the selection of the feed rate. The result of froth overload is an increase in air holdup because mobility of the particle-bubble aggregates is severely restricted. In fact, there is the tendency for the loaded bubbles to be carried downwards towards the tailings if the feed rate is too high. In the situation where this occurs, the run may have to be stopped. In the column used in these tests, which is equipped with manometers for air holdup measurements, this will be indicated by a drop in level in the lower manometer and in the worst case the coal laden bubbles would be seen in the lower manometer where there is very little downward flow. This condition has been referred to as an *inversion*. The rise velocity of the heavily laden bubbles is low and thus are easily carried downward by the moving fluid. The effect is reduced by addition of frother.

Effect of air

The correlation also shows that the yield is a function of the interaction between the wash water and air flow rates. While Coffin and Miszckak (1982) have reported that recovery could be correlated with air flow rate, Dobby *et al*. (1985c) found a relationship between air holdup and recovery. Amelunxen (1985) has stated that a better correlation exists between air holdup and recovery than between air flow rate and recovery.

The results show a decrease in yield with increasing air flow. As air flow rate increases the total bubble surface area increases and the yield should increase. Also, as air flow rate increases, the average bubble diameter increases, resulting in a reduction in the specific bubble surface area and hence a reduction in yield. It appears therefore that there exist two opposing effects, the first dominating at low air flow rate and the second at high air flow rate. This suggests an optimum intermediate air flow at which the best yield is expected. Similar results have been observed by Laplante *et al.* (1983) and Dobby *et al.* (Jan. 1985). The implication here is that the air flows used in these tests are on the high side where yield drops with increasing air flow. This was clearly shown in the backmixing and residence time distribution tests of Chapter 4. A close look at the test results show that there is a slight increase in product ash with increased air flow rate but the increase is minimal because the two air flow rates are not significantly different. The higher limits of air flow rates were required to provide reasonably high froth mobility and reduce froth 'freeze'.

Effect of Wash Water Flow Rate

From the analysis of the results of product ash of test run 1 to 14, given in Appendix B, it appears that the washing effect of the wash water on the product ash levelled off after a wash water flow rate of about 0.75 L/min (0.4cm/sec.

superficial velocity). In some cases there was a decrease and then an increase in the product ash. The only plausible explanation would be that the higher wash water flow rate had a non-selective shearing effect on all particles. This effect might have been enhanced by the near bubble overload condition that existed, by virtue of the high floatability of the coal used and the small percentage ash present. This result was unexpected so further testing was deemed necessary.

The second set of tests was designed specifically to investigate the effect of wash water addition rate on product yield, percent ash and percent solids. In runs 15 to 18 a higher ash feed was used (30%). In addition, froth samples along the column were taken using a vacuum system. The results are given in Table 5.3 to 5.6. None of the above observations regarding the levelling off of the ash after some high wash water rate is apparent. The product ash decreases significantly but not dramatically with increasing wash water addition rate as shown in Figure 5.5 but this happens at the expense of coal yield which drops off with increasing wash water as shown in Figure 5.6. This means that a compromise must be reached between low product ash and yield to select the appropriate wash water rate. It is also noted that the product percent solids decreases with increasing wash water rate. This is shown in Figure 5.7. This means that some wash water returns to the concentrate and the amount that returns depends on the wash water flow

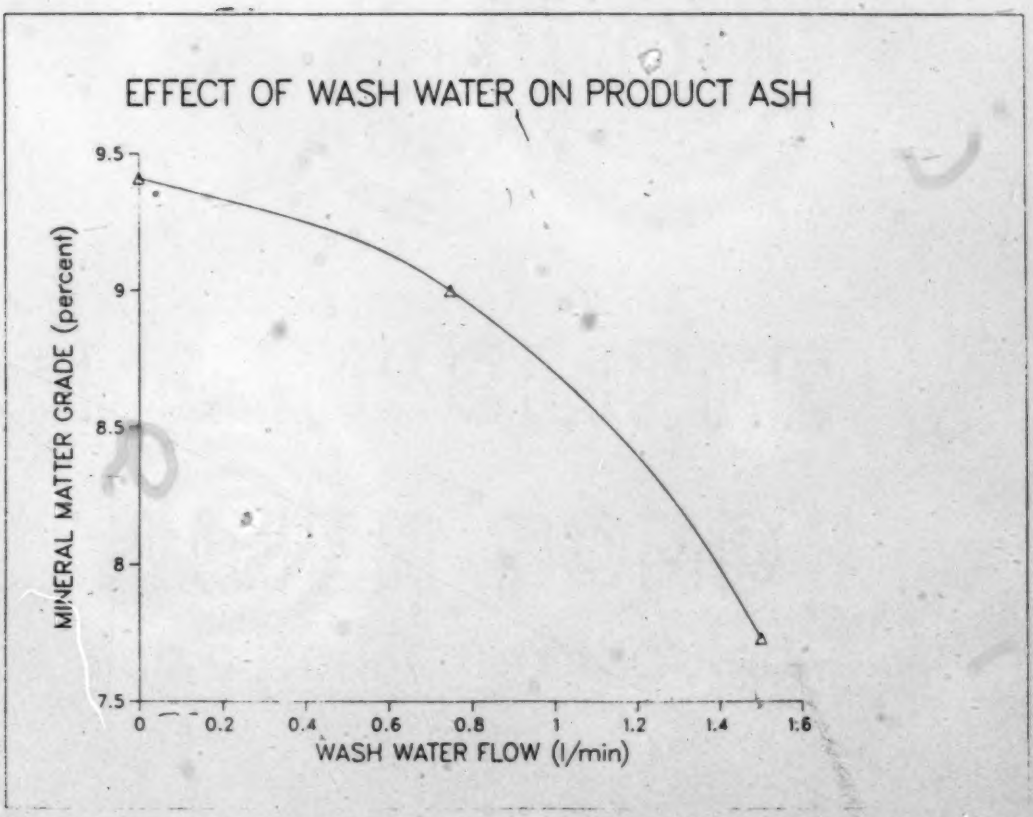


Figure 5.5 Effect of Wash Water Addition Rate on Product Ash

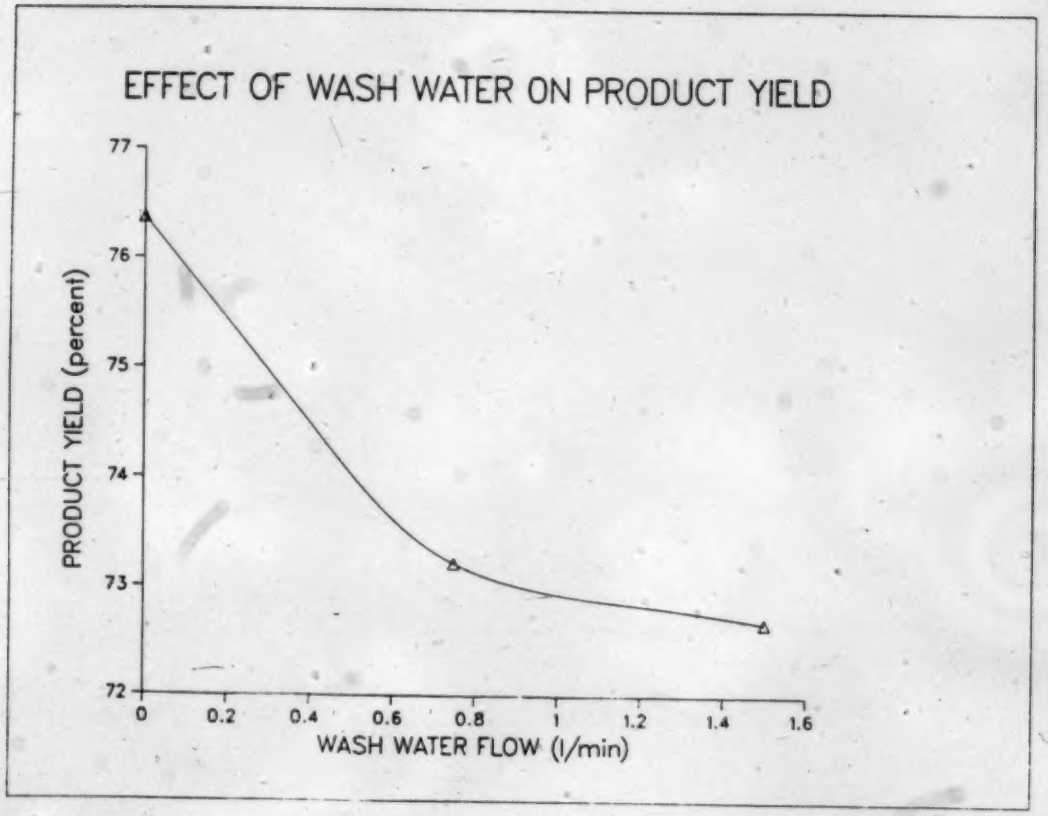


Figure 5.6 Effect of Wash Water Addition Rate on Product Yield

rate. The implication is that there is water exchange between the downward flowing stream and the upward flowing stream and that the transfers depend on the relative amounts of these streams. This water exchange is responsible for keeping entrained feed liquid entering the product to a minimum.

Conclusion

The following conclusions can be made. Feed flow rate does not appear to have significant effect on product yield under the conditions used in this study. It must however be set to provide the required residence time.

There is water exchange between the wash water and the entrained slurry that limits the amount of slurry entering the product and this exchange depends on the relative amounts of wash water and entrained slurry.

It appears that in the event of near froth overload operations, high wash water addition may result in non-selective shearing of particles, resulting in higher product ash and lower solids recovery.

In most cases, flotation is applied to coals that are difficult to clean. For these materials the column is ideal. When column flotation is applied to highly floatable coals the potential for froth overload is greatly enhanced. Froth removal may become the rate determining step and higher frother consumption could result.

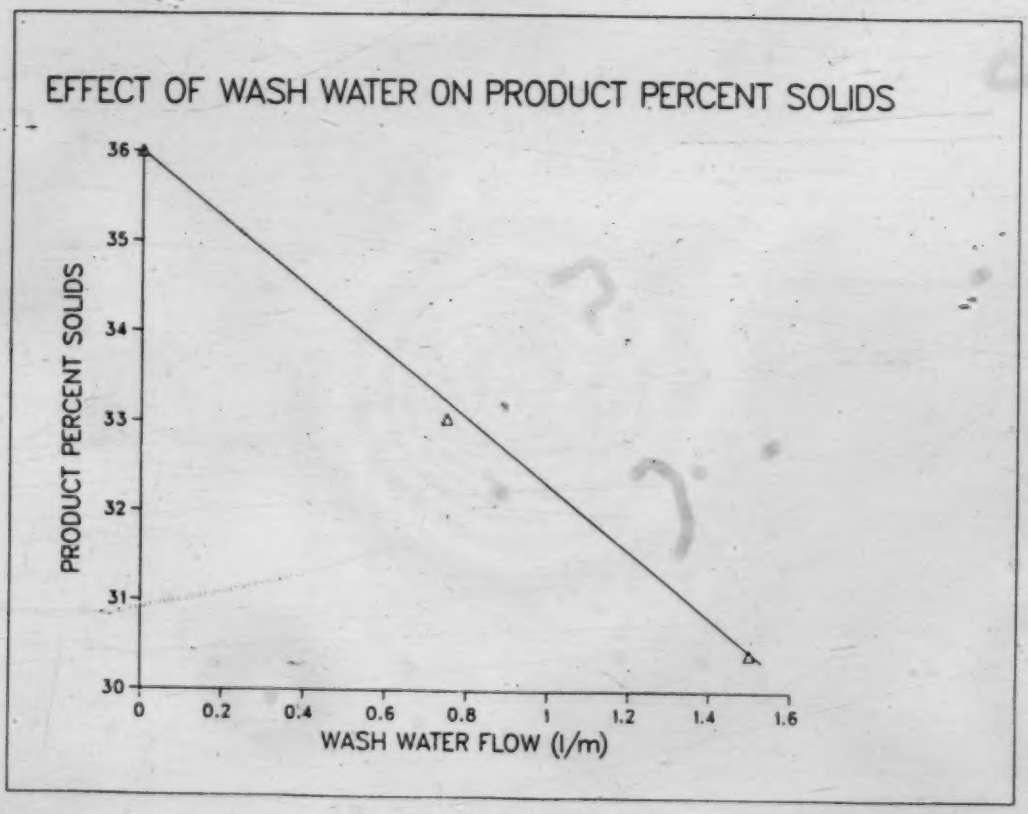


Figure 5.7 Effect of Wash Water Addition Rate on Product Percent Solids

## 5.6 Model Verification

The procedure followed in obtaining the model solution is outlined in section 5.4 of this chapter. There are many parameters, giving an indication of the complexity of the flotation froth and how much more has to be learned about it. Independent estimates of as many of them as possible is recommended.

The gas holdup fraction at each level was estimated from equation 5.18. The thickness of slurry carried on the bubbles $\delta$ was estimated by integration of equation 5.21. Initial values were obtained such that the estimated volume fraction of liquid in the base of the froth column was the same as found in practice. The rate coefficients into the froth were combined with the bubble residence time in equations 5.7 and 5.9 and obtained from experimental data. The rate coefficients between the subsystems were adjusted to simulate the experimental mineral matter data.

Figure 5.8 shows the ash data from runs 15, 17 and 18 which differed only in the wash water flow rate. Figure 5.9 shows the simulated data from the model. The parameters for the model were as follow: $\delta$ =0.006 cm, at the froth/pulp interface, rate coefficients for attached coal, entrained coal, entrained mineral matter were respectively, 0.0005 $s^{-1}$, 0.0025 $s^{-1}$, and 0.003 $s^{-1}$. The rate coefficient for reattachment was 0.0001 $s^{-1}$. The parameters nf and kv for the hydraulic model were 560.0 and 0.8 respectively for 0.75 l/min. wash water. A computer program in fortran used in

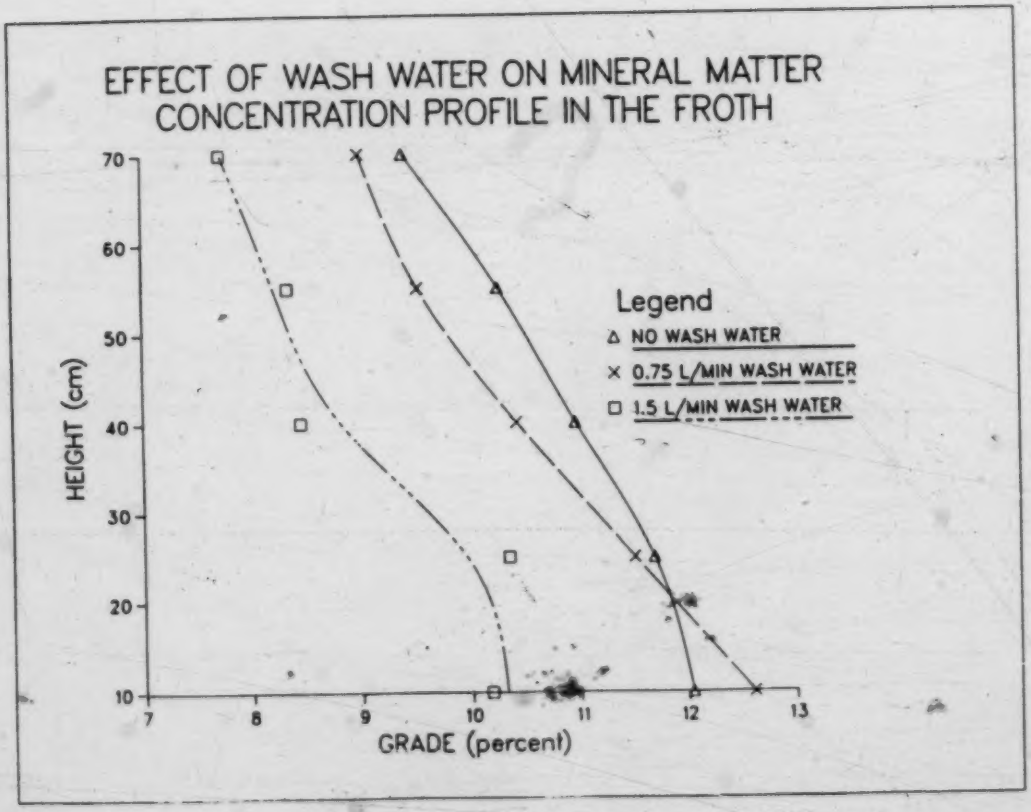


Figure 5.8 Effect of Wash Water on the Mineral Matter Concentration Profile in the Froth

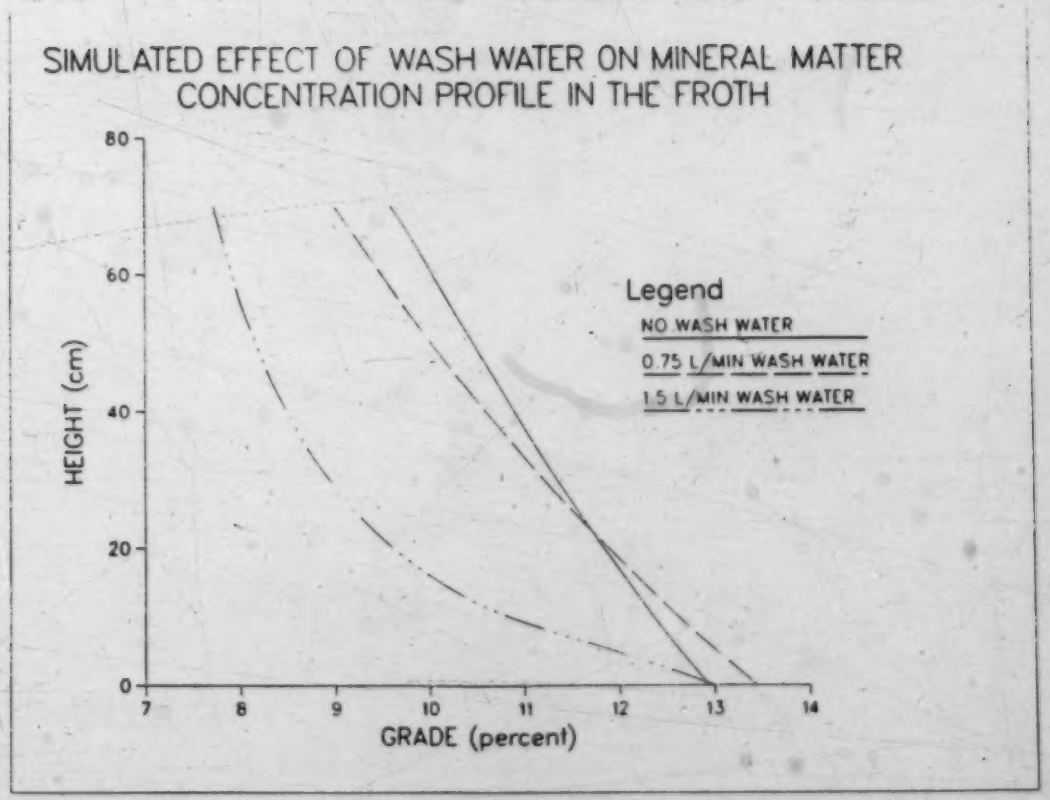


**Figure 5.9 Simulated Effect of Wash Water on the Mineral Matter Concentration Profile in the Froth**

obtaining the model solution is presented in Appendix C. The simple Euler method was used in the numerical integration.

Figure 5.10, 5.11 and 5.12 shows the comparison of the simulated and observed mineral matter profile for each of runs fifteen, seventeen and eighteen.

These figures show fairly good agreement. The model predictions are acceptable on the basis of a Chi-squared test. When there is no wash water, the variation of ash with froth height is close to linear. As wash water flow rate is increased, the the curvature increases. The estimation of percent solids was fairly good but less so than the mineral matter estimation. Figure 5.13 shows a comparison of the observed and simulated percent solids profile for run seventeen. The difficulty of sampling the froth may account for some of the deviation due to error in the experimental data. This is especially true for the percent solids measurement. The addition of wash water some distance below the surface of the froth tends to make the percent solids lower in that region than would be expected. The present understanding of water transfer processes upon which the formulation of the hydraulic model was based may not be entirely adequate. The model of Hartland and Barber which was modified and used here was derived for foams and does not consider the effect of solids on the water transfer.

Nevertheless the model is good in explaining the washing action of the wash water. It takes into account most of the subprocesses observed in the froth. It gives a good

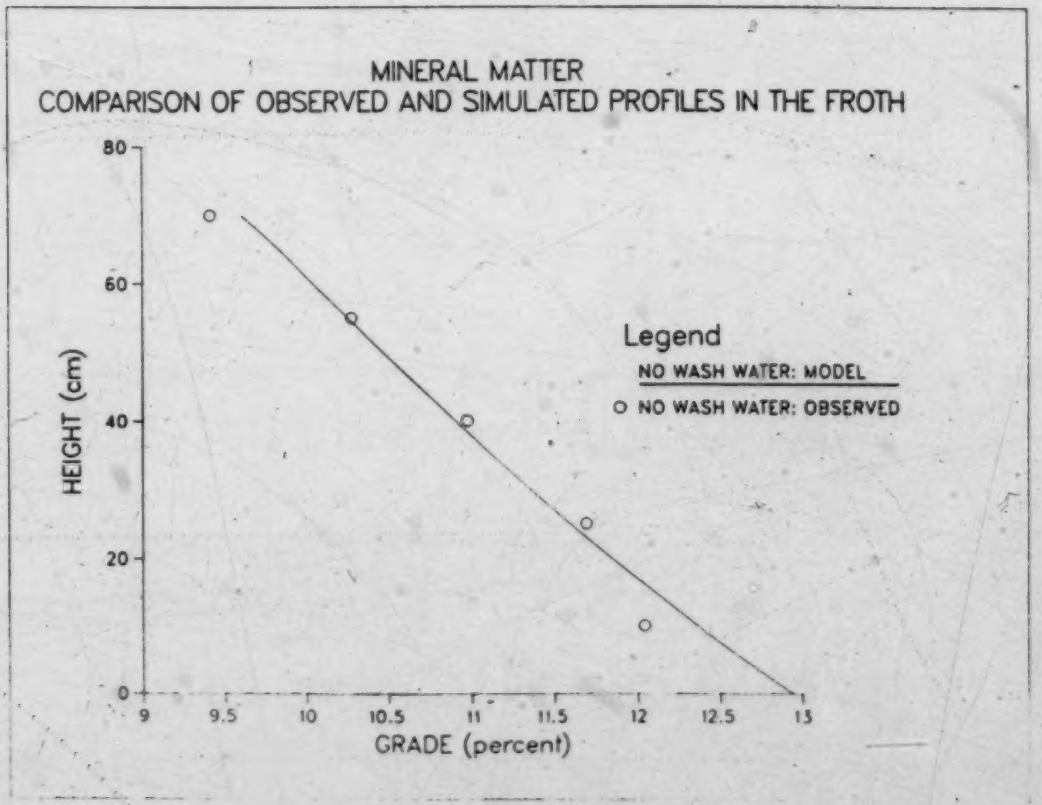


Figure 5.10 Comparison of Simulated and Observed Mineral Matter Concentration Profile in the Froth for no Wash water

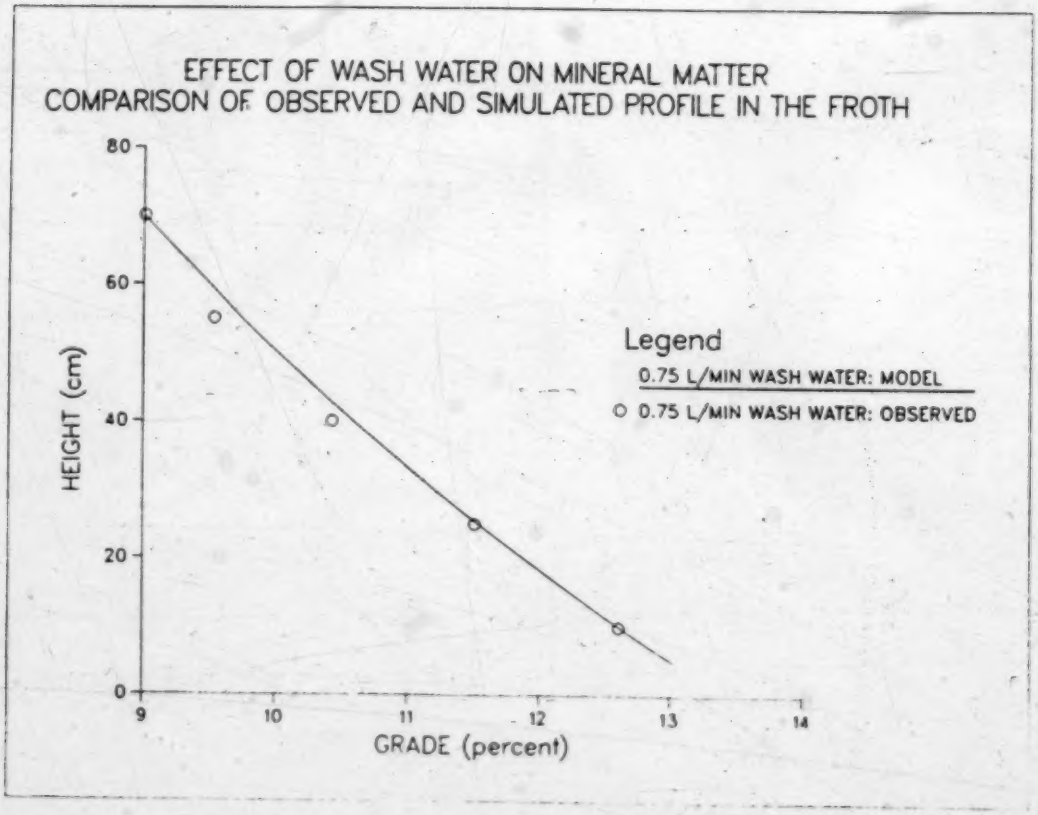


Figure 5.11 Comparison of Simulated and Observed Mineral Matter Concentration Profile in the Froth for 0.75 l/min. Wash water

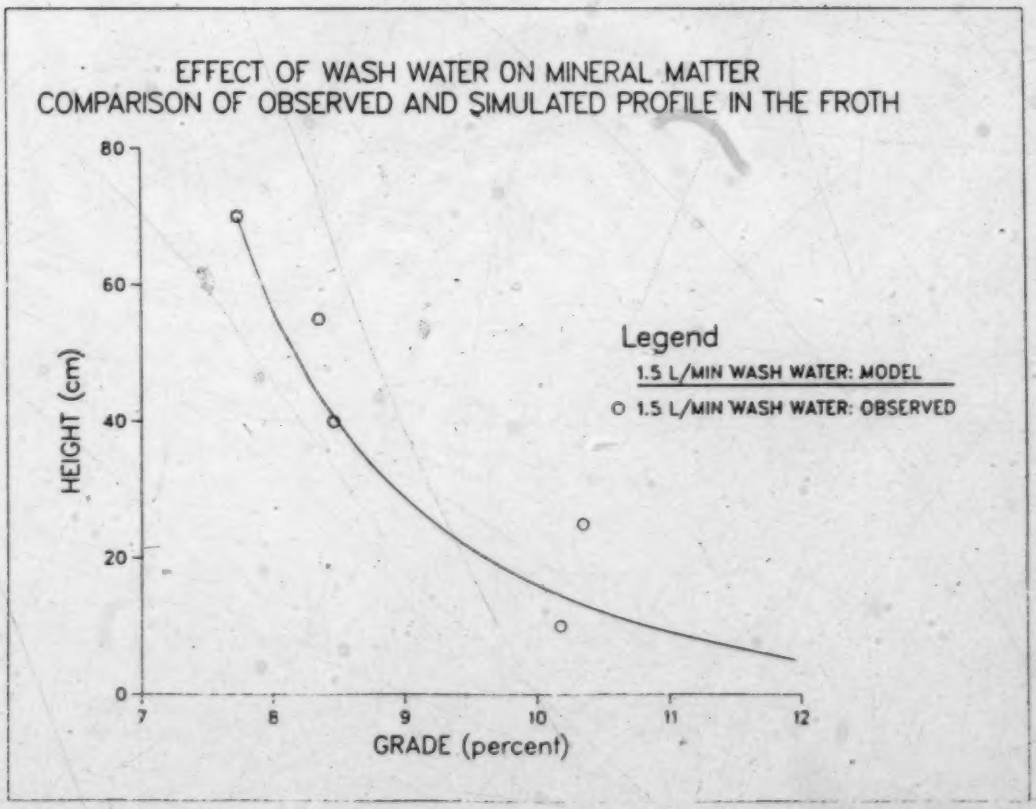


Figure 5.12 Comparison of Simulated and Observed Mineral Matter Concentration Profile in the Froth for 1.5 l/min Wash water

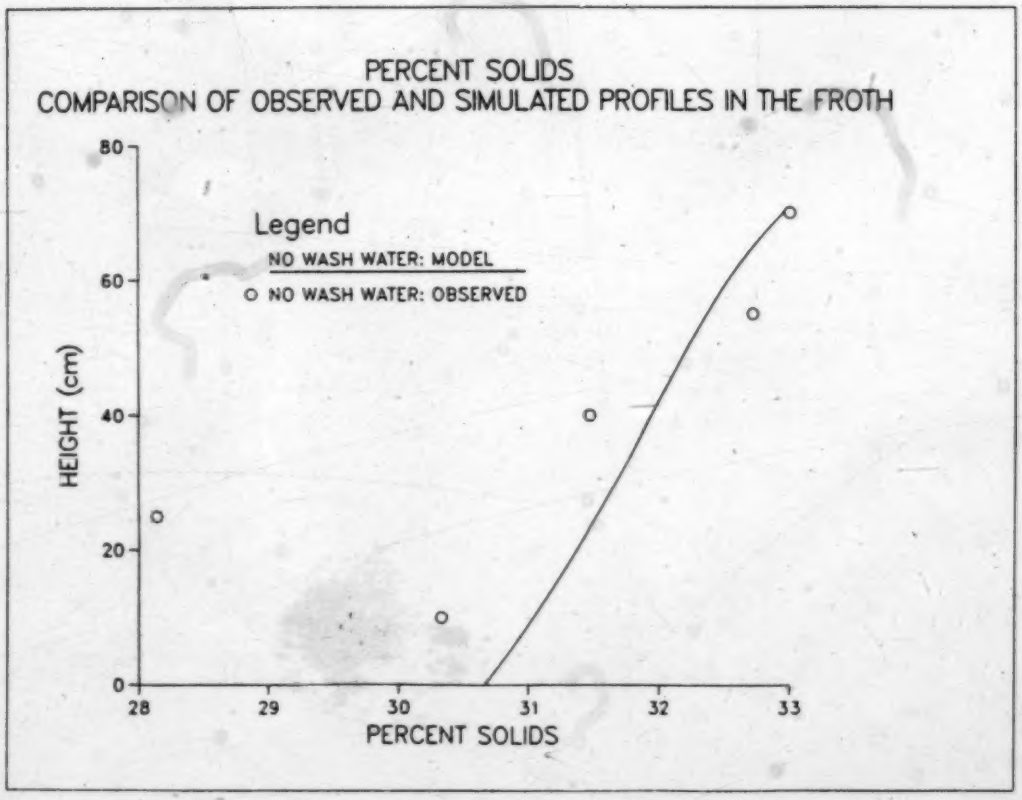


Figure 5.13 Comparison of Simulated and Observed Percent Solids Profile in the Froth for no Wash water

understanding of the behaviour of the froth, both washed and unwashed and should provide the basis for further development and perfection.

## 6. SUMMARY AND CONCLUSIONS

A moderately sophisticated laboratory flotation column has been set up at the University of Alberta for the development of a flotation froth model and, in the medium term, for the study of column flotation of Western Canadian coals in the countercurrent system. The flow dynamics in a continuous flow system such as the flotation column is known to affect the performance of such systems and therefore a detailed study of the flow characteristics was undertaken. This study was done in terms of direct flow visualization, experimental investigation of backmixing and residence time distribution. It was established that some degree of dispersion exists in the laboratory flotation column which is mostly induced by the counter-current flow of air. The degree of dispersion is not considered large enough to have a significant detrimental effect on the operating performance of this column at the operating conditions being used. In a larger system, baffling may be necessary to reduce dispersion. No appreciable stagnant volume was observed. The residence time distribution model that closely describes the flow characteristics in this column is the fractional tubularity model, which is a combination of plug flow and perfect mixers in series.

The column was found to be suitable for coal flotation at appropriate operating conditions from coal flotation studies done with this column. For highly flotable coals, there exists a potential for froth overload, a condition

which is thought to be detrimental to flotation . The column is particularly suitable for cleaning the moderate to difficult to clean coals.

Recommended improvement in the laboratory flotation column set up is as follows:

1. Another sparger should be installed half-way up the collection zone to increase the air throughput without an excessive increase in pressure drop across the sparger. This would ensure that a bubbly flow regime is maintained while higher air flow rates are used.
2. An automatic level controller would increase the ease of operation of the system by reducing operator attention.
3. Different wash water distribution systems should by tried to obtain a more effective system.

The studies show that the column has a potential of providing an effective way of cleaning coals that are difficult to clean. Recommended further studies should include the following:

1. Pilot scale studies to assess the potential of a large scale industrial application and to study the problems associated with scale up of laboratory results to pilot or industrial scale operations.
2. The scale-up studies should be coupled with research to develop the appropriate technology for generating fine bubbles for pilot and large scale columns. This is known to be a major source of concern in operating columns.
3. It is also felt that a joint effort by the University

and the Mining companies is the surest way of establishing the full potential of this new technology and the technology transfer, if it is found appropriate. The University should therefore solicit the cooperation of the mining companies to participate in the research into the technology.

The characterization of the flotation column froth was also the subject of this study. After a thorough literature review and laboratory study of the froth column, a realistic phenomenological model has been proposed to describe the mass transfer occurring in the froth. It takes into account particle transfer by entrainment, detachment and reattachment in the froth. It also includes a water exchange model which describes water transfers within the froth. The approach used to develop the model is probably the most appropriate way of describing the distributed nature of the froth. The model gives reasonably good predictions of the grade profile along the froth length. There still remains some work to be done to improve the prediction capabilities of the model. The current knowledge of the froth behaviour is still limited and the rate coefficients have to be determined from experiments. The model contains many parameters and so some of them have to be estimated independently.

The model is a significant contribution to the current knowledge of the froth. The proposed model in this study should form the basis for further study and refinement to

explain the cleaning action of the froth, both washed and unwashed. Future efforts could appropriately be focussed on the following:

1. The study of the froth to include phenomena like bubble breakage and coalescence into the model. Of course, inclusion of these and other subprocesses in the model will increase its complexity unless those processes are well understood. Accumulation of knowledge about these subprocesses through study such as that reported in this thesis will help determine those with significant impact on the overall process.
2. The establishment of correlations for the rate constants in the model. Once good correlations are established for these coefficients, the cleaning zone model could be interfaced with existing collection zone models to predict the overall flotation performance.

Phenomena like column drainage observed by some researchers were not explicitly included into this model.

Froth modelling is a relatively unsophisticated yet essential component of flotation modelling and more research is required in that area.

## Bibliography


Amelunxen, R.L., "The Mechanics of Operation of Column Flotation Machine", Paper Presented at The 17th Canadian Mineral Processors conference, Ottawa, Jan. 1985.

Anon., "Flotation Column due for Mill Scale Tests", Eng. and Mining J., 166(1),1965, pp. 76-83.

Arbiter, N., "Flotation Rate and Flotation Efficiency", Trans. AIME, Vol 190, 1951, pp. 791-796, 1951.

Arbiter, N., and Harris, C.C., "Flotation Kinetics", Froth Flotation,Fuerstenau, D., (ed), AIME, New York, 1962 pp. 215-262.

Bascur, O. R., PhD. Thesis, University of Utah, 1982.

Bascur, O. R., Herbst, J. A., "Dynamic Modeling of a Flotation Cell With a View Towards Automatic Control", XIV Inter. Min. Proc. Congress, Oct. 1982, pp. III-11.1 - III-11.22.

Bikerman, J. J.," Foams", Springer-Verlag, New York, Heidelberg, Berlin, 1973.

Boutin, P., Wheeler, D.A., "Column Flotation," Mining World, Vol 20 No 3 1967, pp. 45.

Boutin, P., Wheeler,D.A.; "Column Flotation Development Using 19-inch Pilot Unit", Can. Min. Journal, Vol 94, March 1967, pp. 94-101.

Bull, W. R., "The Rates of Flotation of Mineral Particles in Sulphide Ores", Proc. Aust. IMM, No 220, 1966, pp. 69-78.

Coffin, V. L., and Miszckak, J., "Column Flotation at Mines Gaspe", Preprint No IV-21, XIVth Int. Min. Proc. Congr., Toronto, Canada, 1982.

Cooper, H. R., "Feedback Process Model of Mineral Flotation", Trans. Soc. Min. Eng., AIME 235, 1968 pp. 439-446.

Cutting, G. W., Barber, S.P., and Newton, S., "Effect of Froth Structure and Mobility on -", Int. J. Miner. Process., 16, 1986, pp. 43-61.

Cutting, G. W., Barber, S. P., Watson, D., "Prediction of Flotation Plant Performance from Batch Tests Using Process Models: Effect of Froth Structure", 14th Int.

Min. Proc. Cong., Toronto, 1982, pp. IV-14.1 - 14.20.

Cutting, G. W., and Devenish, M., "A Steady State Model of Froth Flotation Structures", S. Min. Eng, AIME, Annual Meeting, New York, 1975, Preprint 75-B-56.

Cutting, G. W., Watson, D., Whitehead, A. and Barber, S. P., "Froth Structure in Continuous Flotation Cells:-", Int. J. Miner.Process., 7, 1980 pp. 347-369.

Desai, D., and Kumar, R., "Flow Through a Plateau Border of Cellular Foam", Chem. Engng. Sc., 9, 37, 1982, pp. 1361-1370.

Dobby, G. S., Finch, J. A., "Flotation Column Scale-up and Simulation", Paper presented at 17th Canadian Mineral Processors Operators Conference. Jan 22-24 1985a.

Dobby, G. S., and Finch, J. A., "Particle Size Dependence in Flotation Derived from a Fundamental Model of the Capture Process", AIME, New York, 1985b.

Dobby, G.S., Amelunxen, R., Finch J.A., "Column Flotation: Some Plant Experience and Model Development", AMRD-IFAC 1985c, pp. 259-265,

Dowling, E.C., Klimpel, R.R., Applan, F.F., "Model Descrimination in the Flotation of Porphery Copper", Paper Presented at the Annual Meeting of AIME, Los Angeles, February 1984.

Ergun, S., "Fluid Flow Through Packed Columns", Chem. Engng. Prog., 48, 2, 1952, pp. 89-94.

Flint, L.R., PhD. Thesis. University of Queensland, 1971.

Foot Jr,D.G.; Private Communication 1985.

Gaudin, A.M., "Flotation", McGraw-Hill, New York, 1934.

Groppo, J., "Column Flotation Shows Higher Recovery with Less Ash", Coal Mining, August 1986, pp. 36-38.

Harris, C.C., and Rimmer, H.W., "Study of a Two-phase Model of the Flotation Process", Trans. IMM. 75, 1966, pp. C153-C162.

Hartland, S., Barber, A.D., "A Model For a Cellular Foam", Trans. Inst. Chem. Engns., Vol. 52, 1974. pp. 43-51.

Himmelblau, D.M., Bischoff, K.B., "Process Analysis and Simulation / Deterministic Systems", John Wiley and Sons, 1968.

Ho, G. E., Prince, R.G.H., "Characteristics of Cellular Foams on Perforated Plates", Proceedings of Symposium on Bubbles and Foams, Varem deutcher Ingenierue, Nurenberg, 1971.

Honeywell,W.R., "Column Flotation of Uranium from Eliot Lake", Mines Branch Canada. EMR Technical Bulletin #TB-96,AP 11 (1967).

Imaizumi, T. and Inoue, T., "Kinetic Consideration of Flotation", Proceeding of Sixth Int. Min. Proc. Con., Cannes, 1963, ed. Roberts, A., Pergamon, Oxford, 1965, pp. 581-605.

Jameson,G.J., Nam, S., and Moo Young, M., Minerals Sci. Engng., July 1977, pp. 103-118.

Kelsall, D.F., "Application of Probability in the Assessment of Flotation Systems", Trans. Inst. Min. Met.,70, 1961, pp. 191-204.

King, R.P., "Pilot Plant Investigation of a Flotation Model", Report No. 1573, NIM, South Africa, August 1972.

Klassen, V.I., and Mokrousov, V.A.,(1959), "An Introduction to the Theory of Flotation", Butterworth, London (Translated from Russian by J. Leja G. W. Poling, 1963)

Laguitton, D., "Simulated Processing of Ore and Coal Manual", Chapter 4, Mineral Science Laboratories Division Report MRP/MSL 82-129(IR) 1982.

Laplante, A.R., Toguri, J.M., and Smith, H.W., "The Effect of Air Flow Rate on The Kinetics of Flotation. Part 1", Int. J. Min. Proc., 11, 1983, pp. 221-234.

Leonard, R. A., and Lemlich, R., "A study of Interstitial Liquid Flow in Foams", AIChE Journal Vol. 11 No. 1, 1965, pp. 18-25 and pp. 25-28.

Levenspiel, O., "Chemical Reaction Engineering", John Wiley & Sons Inc. 2nd Ed., 1972, pp. 286-287.

Loveday, B.K., and Marchand, G.R., "Simulation of Multi-component Flotation Plant", Journal of South African Inst. Min. Met., 1972 pp. 288-294.

Lutterell, G. H., Adel, G.T., and Yoon, R.H., "Modeling of Column Flotation", Preprint No. 87-130, Presented at SME Annual Meeting, Denver, Colorado, Feb. 24-27, 1987.

Lynch, A. J., Johnson, N.W., Manlapig, E.V., and Thorne C.G "Mineral and Coal Flotation Circuits; Their Simulation and Control", Fuerstenau, D.W., Advisory Editor, Vol. 3

Elsevier, New York,1981.

Maksimov, I. I., and Khainman, V. "Effect of Processes Occurring in the Froth on the Rate and Selectivity of Flotation", Soviet J. Non-Ferrous Metals, 6 No. 5, 1965 pp. 8-10.

Mathieu, G.I., "Comparison of Flotation Column with Conventional Flotation for Concentration of a Molybdenum Ore", CIM Bulletin, May 1972, pp. 41-45.

Mckay, J.D., Foot Jr, D.G., Huiatt, J.L., "Column Flotation of Montana Chromite Ore", Minerals and Metallurgical Processing. Aug 1986. pp. 170-177.

Mecklenburgh, J.C., Hartland, S., "The Theory of Backmixing", John Wiley & Sons, Ltd., 1975.

Mika, T., "Discussion of Paper by Cooper, H.R.", Trans. Soc. Min. Eng. AIME 238, 1967, pp. 479-483.

Moon, K.S.,"Counter-current Column Flotation Machine", project ERP- 4.3.03.02. Energy Research Program, Mineral Sceinces Laboratories, Division Report EPR/MSL 82-55(IR) 1982.

Moon, K.S. and L.L. Sirois, "Beneficiation of High Sulphur Coal by Column Flotation", CANMET Report #ERP/MSL 82-67 (IR) 1982, pp.34.

Moys, M. H., "A Study of a Plug Flow Model for Flotation Froth Behaviour", Int. J. Miner. Process., 5, 1978, pp. 21-38.

Moys, M. H., "Residence Time Distribution and Mass Transport in the Froth Phase of Flotation Process",Int. J. Miner. Process., 13, 1984, pp. 117-142.

Narashimhan, K.S., etal,"Column Flotation Improves Graphite Recovery", Eng.Min.J. (May 1972).

Nauman, E.B., and Buffham, B.A., "Mixing in Continuous Flow Systems", John Wiley and Sons, 1983, pp. 56.

Nicol, S.K., Roberts,T., Bensley, C.N., and Lamb, R., "Column Flotation of Desliming Cyclone Overflow for the Riverside Coal Preparation Plant", Preprint No 87-92, Paper presented at SME Annual Meeting, Denver, Colorado Feb. 1987.

Rice, R.G., Oliver, A. D., Newman, J.P., and Wiles, R. J., "Reduced Dispersion Using Baffles in Column Flotation", Powder Technology, 10, 1974 pp. 201-210.

Rogers, R.S.C., and Gardiner, R.P., "Use of a Finite-Stage Transport concept for analyzing RTD of Continuous Processes", AIChE J. 25, 1979, pp. 229-240

Santanam, C.B., and Tietbohl, G.L., "Complex Flow Visualization by a Unique Technique", Flow Visualization III, Papers Presented at the Third International Symposium on Flow Visualization, Ann Arbor, 1983, pp. 1-6.

Satry, K.V.S., "Processing of Mineral Fines by Column Flotation", Perfomance Report, U.S. Department of interior State Mining and Mineral Resource Research Inst. 1982.

Sastry, K.V.S., and Fuerstenau, D.W., "Theoritical Analysis of the a Countercurrent Flotation Column", Trans. SME, AIME vol. 247, March 1970, pp. 46-52.

Schuhman, R., "Method of Study State Study of Flotation Problems", J. Physical Chem. 46, 1942, pp. 801-902.

Schulze, H. J., "Physico-chemical Elementary Processes in Flotation", Developments in Mineral Processing 4, Elsevier, 1984, pp. 19.

Shah, Y.T., Stiegel, G.J., and Sharma, M.M., "Backmixing in Gas-Liquid Reactors", J. of AIChE vol. 24, No. 3, May 1978, pp. 369-400.

Steiner, L., Hunkeler, R., and Hartland, S., "Behaviour of Dynamic Cellular Foams", Trans. IChemE, vol. 55, 1977, pp. 153-163.

Sutherland, K.L., "Physical Chemistry of Flotation", J. of Phys. Chem. 52, 1948 pp. 394-425.

Tomlinson, H.S., and Fleming, M.G., "Flotation Rate Studies, Mineral Processing", (ed. A. Roberts), 6th Int. Min. Proc. Congress, Cannes, 1965, pp. 563-579.

Taggart, A., "A Handbook of Ore Dressing", (2nd edition), John Wiley & Sons, New York, 1942.

Venkataraman, K.S., Fuerstenau, D.W., "Effect of Lifter Shape and Configuration on Material Transport, Part 1, In Rotating Drums", Powder Technology. 1986.

Wehner, J.F., and Wilhelm, R.H., Chem. Eng. Sci.,6, 89 1956.

Wen, C.Y., and Fan, L.T., "Models For Flow Systems and Chemical Reactors", Marcel Dekker Inc. 1975, pp 482.

Wheeler, D.A.; "Big Flotation Colunm Mill Tested", Eng Min

J. Nov 1966, pp. 98-103.

Xu, Changlian, "Kinetic Models for Continuous Flotation in Flotation Columns", XVth Int. Min. Proc. Cong., Cannes, 2-9 June 1985.

Yianatos, J.B., Finch, J.A, and Laplante, A.R., "Holdup Profile and Bubble Size Distribution of Flotation Column Froths", 24th Annual Conf. of Met., 1985, Vancouver, (CIM).

# Appendix A

## Feed and Wash Water Flow Meter Calibration

Calibration of Flow Meters

The magnetic flow meter on the feed line and the rotameter were both calibrated in litres per minute.

Magnetic Flow Meter Calibration

The magnetic flow meter was calibrated by setting an adjustable resistor in the signal convertor to the desired value. The calibration factor of the meter is given as 14.16 U.S gallons/min by the manufacturer. The appropriate resistance is obtained from the following relationship:

$$\text{Resistance} = 40{,}000 \times \frac{\text{Flow at 100\%(20 mA)}}{\text{Cal Factor}}$$

The maximum flow that was required was 8.0 l/min or 2.11 USGPM. Substitution of 2.11 in the above relationship yields a resistance of 5960 ohms. The resistor was adjusted to this value using a multimeter. After this calibration was completed, the measurements were checked by taking samples of running water through the flow meter for a given time. A plot of the calibration curve is given in Figure 6.1.

Wash water Flow Calibration

The wash water flow meter was calibrated by taking samples of water running through the flow meter for specified times at different settings in percentage of maximum setting of the flow meter. A plot of the measured flow rate against the percentage of maximum setting is shown in Figure 6.2. An equation was fitted to the calibration data points as

$$\text{Flow(l/min)} = 0.1318 \times (\%\text{Reading}) - 2.2$$

which could be used to determine the setting for a given flow.

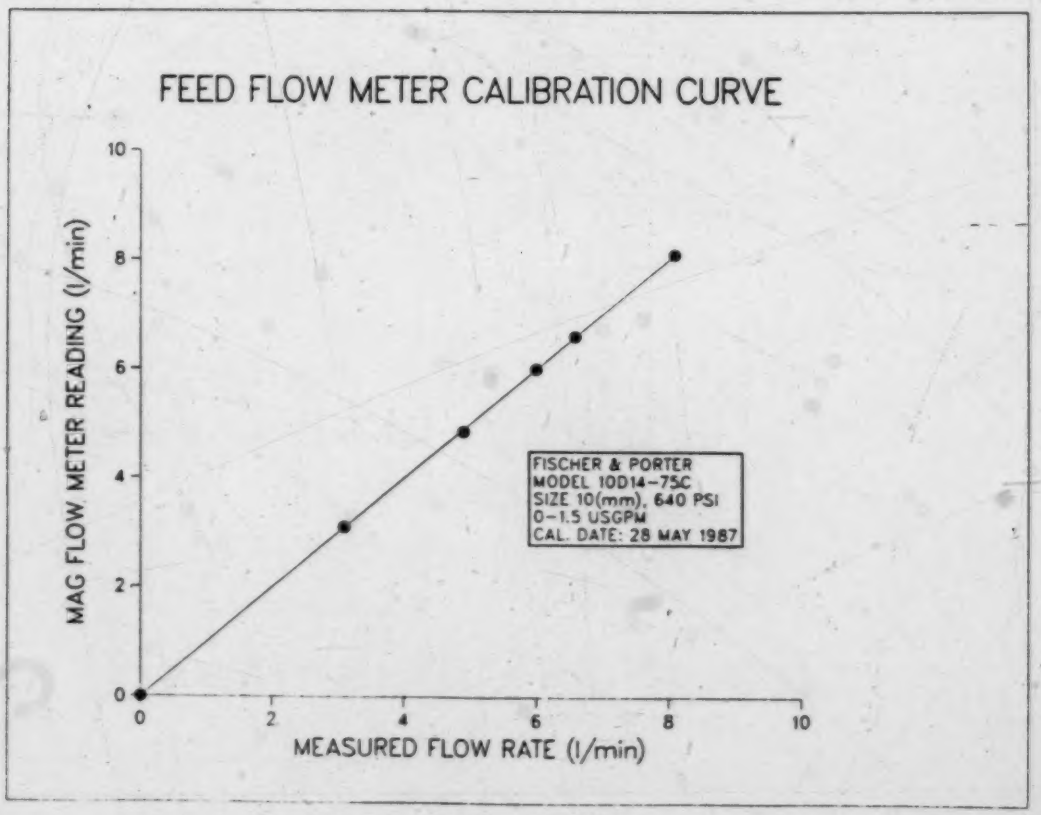


Figure 6.1 Magnetic Flow Meter Calibration Curve

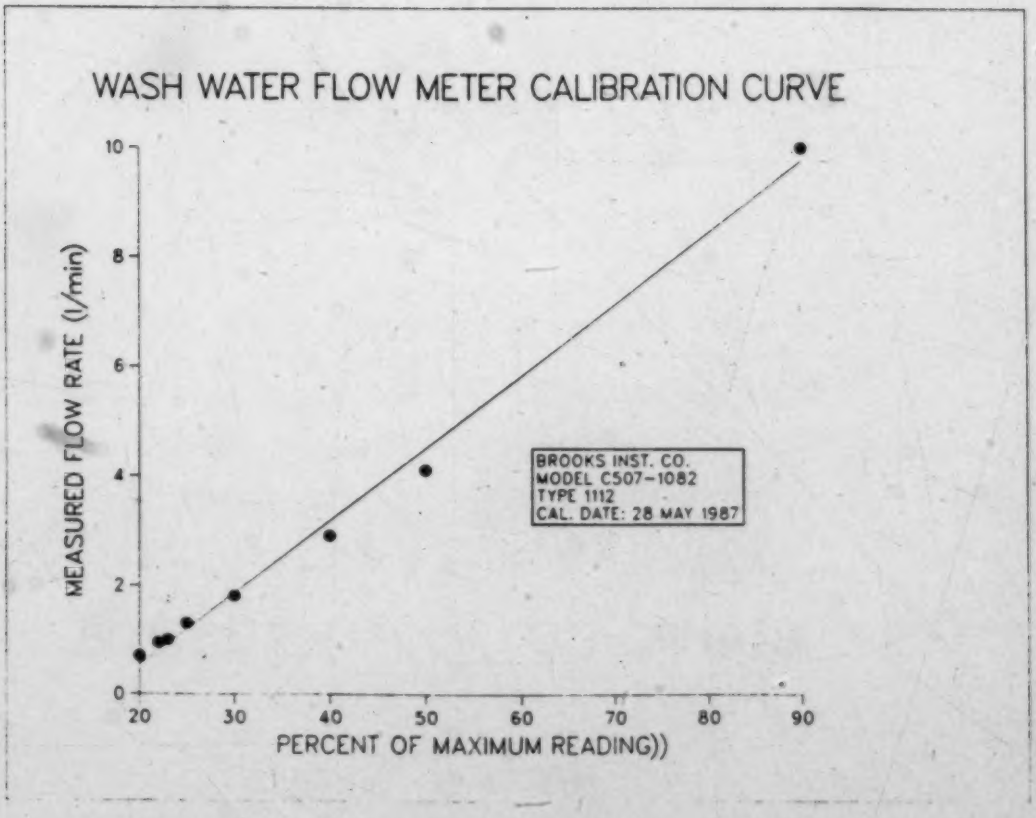


Figure 6.2 Wash Water Flow Meter Calibration Curve

# Appendix B

## Experimental Data For Runs 1 To 14

Table 6.1 Column Flotation Results for Run 1

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 2.8 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 0.0 | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 1.5 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 12.38 | 15.66 |
| 2 | 10.42 | 30.65 |
| 3 | 10.55 | 34.87 |
| 4 | 10.10 | 31.70 |
| 5 | 9.29 | 33.04 |
| 6 | 9.18 | 32.75 |
| 7 | 91.07 | 0.39 |

Product Yield = 86.79%

Table 6.2 Column Flotation Results for Run 2

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 3.5 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 0.0 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 1.5 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 13.31 | 13.74 |
| 2 | 10.13 | 32.60 |
| 3 | 9.89 | 29.34 |
| 4 | 9.55 | 29.55 |
| 5 | 10.06 | 30.38 |
| 6 | 8.79 | 33.68 |
| 7 | 91.90 | 0.35 |

Product Yield = 86.55%

Table 6.3 Column Flotation Results for Run 3

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 2.8 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 0.0 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 12.76 | 15.28 |
| 2 | 10.17 | 32.52 |
| 3 | 9.67 | 31.92 |
| 4 | 9.50 | 31.97 |
| 5 | 10.80 | 28.83 |
| 6 | 8.69 | 35.20 |
| 7 | 92.84 | 0.33 |

Product Yield = 86.56

Table 6.4 Column Flotation Results for Run 4

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 3.5 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 0.0 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 14.33 | 14.15 |
| 2 | 11.68 | 30.94 |
| 3 | 11.25 | 31.08 |
| 4 | 9.68 | 31.79 |
| 5 | 10.00 | 31.61 |
| 6 | 9.06 | 35.08 |
| 7 | 90.79 | 0.40 |

Product Yield = 86.60%

Table 6.5 Column Flotation Results for Run 5

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 2.8 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 0.75 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 1.5 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 13.03 | 13.47 |
| 2 | 9.85 | 20.45 |
| 3 | 9.52 | 19.86 |
| 4 | 9.31 | 19.73 |
| 5 | 8.96 | 13.10 |
| 6 | 8.20 | 27.35 |
| 7 | 90.68 | 0.28 |

Product Yield = 85.69%

Table 6.6 Column Flotation Results for Run 6

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 3.5 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 0.75 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 1.5 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 12.40 | 15.62 |
| 2 | 9.92 | 19.14 |
| 3 | 9.65 | 18.58 |
| 4 | 9.20 | 18.51 |
| 5 | 9.20 | 18.05 |
| 6 | 7.67 | 24.55 |
| 7 | 91.80 | 0.33 |

Product Yield = 85.30%

Table 6.7 Column Flotation Results for Run 7

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 2.8 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 0.75 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 12.69 | 13.61 |
| 2 | 11.89 | 23.30 |
| 3 | 11.08 | 23.01 |
| 4 | 9.51 | 20.67 |
| 5 | 8.76 | 16.38 |
| 6 | 8.23 | 24.59 |
| 7 | 91.07 | 0.39 |

Product Yield = 85.89%

Table 6.8 Column Flotation Results for Run 8

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 3.5 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 0.75 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 14.61 | 11.97 |
| 2 | 11.40 | 19.85 |
| 3 | 10.44 | 19.76 |
| 4 | 9.60 | 19.95 |
| 5 | 9.75 | 19.17 |
| 6 | 8.41 | 28.97 |
| 7 | 91.07 | 0.39 |

Product Yield = 85.85%

Table 6.9 Column Flotation Results for Run 9

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 2.8 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 1.5 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 1.5 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 12.95 | 11.32 |
| 2 | 9.30 | 15.23 |
| 3 | 9.25 | 14.36 |
| 4 | 9.74 | 13.19 |
| 5 | 9.00 | 13.29 |
| 6 | 8.20 | 24.49 |
| 7 | 92.69 | 0.25 |

Product Yield = 86.03%

Table 6.10 Column Flotation Results for Run 10

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 3.5 L/Min | Feed Ash: | 20% |
| Wash water Rate: | 1.5 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 1.5 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 14.22 | 10.83 |
| 2 | 11.25 | 14.88 |
| 3 | 10.47 | 14.74 |
| 4 | 9.97 | 13.26 |
| 5 | 10.43 | 14.00 |
| 6 | 8.99 | 23.73 |
| 7 | 92.83 | 0.25 |

Product Yield = 86.90%

Table 6.11 Column Flotation Results for Run 11

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 2.8 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 1.5 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 12.93 | 12.57 |
| 2 | 10.27 | 17.09 |
| 3 | 10.07 | 16.44 |
| 4 | 9.29 | 16.75 |
| 5 | 8.88 | 14.56 |
| 6 | 8.10 | 29.62 |
| 7 | 85.70 | 0.27 |

Product Yield = 84.66%

Table 6.12 Column Flotation Results for Run 12

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 3.5 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 1.5 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 13.84 | 10.47 |
| 2 | 11.41 | 14.06 |
| 3 | 11.12 | 13.67 |
| 4 | 9.71 | 13.69 |
| 5 | 10.38 | 14.14 |
| 6 | 9.00 | 23.85 |
| 7 | 87.76 | 0.28 |

Product Yield = 86.03%

Table 6.13 Column Flotation Results for Run 13

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 3.5 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 0.0 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 1.5 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 12.76 | 13.91 |
| 2 | 10.49 | 31.84 |
| 3 | 10.30 | 32.74 |
| 4 | 9.93 | 31.40 |
| 5 | 10.08 | 28.48 |
| 6 | 9.00 | 33.80 |
| 7 | 93.64 | 0.24 |

Product Yield = 87.00%

Table 6.14 Column Flotation Results for Run 14

1. CONDITIONS

| | | | |
|---|---|---|---|
| Feed Rate: | 2.8 L/Min | Feed Ash: | 20% |
| Wash Water Rate: | 1.5 L/Min | Collector: | 3.7 cc/kg Feed Solids |
| Air Flow Rate: | 2.0 L/Min | Frother: | 0.6 cc/kg Feed Solids |

2. DATA

Sample Identification: #1 collection zone sample #2 10 cm above interface
#3 25 cm above interface #4 40 cm above interface
#5 55 cm above interface #6 concentrate
#7 tailings

| SAMPLE # | ASH | % SOLIDS |
|---|---|---|
| 1 | 16.75 | 13.26 |
| 2 | 10.93 | 15.96 |
| 3 | 10.75 | 16.63 |
| 4 | 10.54 | 16.70 |
| 5 | 10.38 | 15.91 |
| 6 | 8.57 | 27.23 |
| 7 | 86.53 | 0.27 |

Product Yield = 85.34%

# Appendix C

## Computer Program for Obtaining Model Solution

```
      PROGRAM  COLUMN
C
C   Program to solve the Flotation Column Froth Model
C         Simple Euler Method is used in the Integration
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C VARIABLE/PARMETER DEFINITIONS:                                       +
C    QA=Air Flowrate              AC=Column x-sectional area           +
C    EG=Fractional air Holdup     DB=Mean bubble diameter              +
C    KD,KA,KDR,KG, KPA1,KPE1,KPE2 are rate constants.                  +
C    CA1,CE1,CE2,CD1,CD2 are solids concentrations  and                +
C    MCA1,MCE1,MCE2,MCD1,MCD2 are solids masses.                       +
C    QW=Wash Water Flow Rate   QF=Slurry flow into froth               +
C    QC=Liquid flow into concentrate.                                  +
C    F=Fraction of froth liquid in films.                              +
C    DEL=Film thickness  G= Gravitational acc'n                        +
C    MEU= fluid viscosity  KV and NF are adjustable model              +
C    parameters.                                                       +
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C
C
      REAL QA,AC,EG,DB,KD,KDR,KG,CA1,CE1,CE2
      REAL MCA1,MCE1,MCE2,QD,KA,KPA1,KPE1,KPE2
      REAL QW,QF,F,DEL
      REAL G,QC,MEU,KV,NF,MCD1,MCD2
C
C   Initialise the parameters.
C   EG0 can be estimated from the
C   modified Hartland foam model or a measured value
C   can be used
C     GAS HOLDUP IN FROTH
      QA=2000.0/60.0
      AC=31.67
      QW=750./60
C     KH=100.0
C     T=2.0
      EG0=0.70
C     QC=KH*6.35*(1-EG0)*T**1.5
      QC=322.11/60.
      KV=0.8
      NF=560.
      G=981.
      DB=0.1
      DELTA0=0.006
      MEU=0.013
C
C     Estimate R0 from equation 5.16
C
      RS1=(1.-EG0)-3.3*EG0*DELTA0/DB
      RS2=DB**2/(1.26*EG0)
      RS3=RS1*RS2
      R0=RS3**.5
      RHO=1.0
      SIGMA=30.0
      WRITE(6,25)
   25 FORMAT( T12,'HEIGHT FROM',T28,'PRODUCT',T40,'PRODUCT',
     +  T52,'PERCENT',/,T13,'INTERFACE',T30,'ASH',T41,
     +  'GRADE',T52,'SOLIDS',//)
```

```
C   Calculate the initial concentration of each species
C
      CA1=0.084
      CE1=0.045
      CE2=0.0495
C
C
      KG=0.003
      KDR=0.0025
      KD=.0001
      KA=.0005
      ALPHAI=.6
      DZ=0.5
      Z=0.
      ZMAX=70.0
      DO 10 H=Z,ZMAX,DZ
C
C With these initial values, solve for the film thickness
C from equation 5.21
C
      DELT1= 29.13*DELTA0**3*SIGMA*AC*EG0
      DELT2=MEU*DB**2*NF**2*R0*QA
      DELT=(DELT1/DELT2)*DZ
      DEL=DELTA0- DELT
C
C Then solve for the gas holdup fraction from equation 5.17
C
      TERM1=113.1*MEU*KV*DB**2/(RHO*G*AC)
      TERM2=3.3*DEL*QA/DB-(QC-QW)
      TERM12=(TERM1*TERM2)**.5
      OTERM=1. + 3.3*DEL/DB
      OT  =1.26/DB**2
      EG=(1 - OT*TERM12)/OTERM
C     PRINT *,EG,OTERM,TERM12,R0
C
C Determine R from equation 5.15
C
      RP1=113.1*MEU*KV*DB**2/(EG**2*RHO*G*AC)
      RP2=3.3*QA*DEL/DB-(QC-QW)
      R4=RP1*RP2
      R=R4**.25
C
C Calculate QF, QD and F
C
      QF=3.3*QA*DEL/DB
      QD1=QF+QW-QC
      QD=0.00845*EG**2*RHO*G*R4*AC/(MEU*KV*DB**2)
      F=(6.*DEL/DB)*(EG/(1.-EG))
      IF(INT(H/5)*5.NE.H)GO TO 24
C     WRITE(6,21)EG,R,DEL,QF,F
C 21  FORMAT(2X,F8.5,2X,F10.6,2X,F12.8,2F8.4)
C
  24  CONTINUE
C
C  Estimate the solids concentration in the downward
C  flowing stream at the pulp froth interface from an
C  overall material balance over the froth column.
C
```

```
117             IF(H.GT.0.0)GO TO 32
118             CMASS=239.84
119             CASH=9.01
120             CCOAL=100.-CASH
121             COALM=CCOAL*CMASS/(3.*60.*100.)
122             ASHM=CASH*CMASS/(3.*60.*100.)
123             QDCD1=QA*CA1+QF*CE1-COALM
124             QDCD2=QF*CE2-ASHM
125             CD1=QDCD1/QD
126             CD2=QDCD2/QD
127             PRINT *,CD1,CD2,QD,EG,F
128       C
129       C  Solve for the solids concentrations
130       C
131          32 DCA1DZ= -AC*EG*KA*CA1/QA+KD*AC*(1.-EG)*(1-F)*CD1/QA
132             DQCE1Z=-AC*(1.-EG)*F*(KDR*CE1)
133             DQCE2Z=-AC*(1.-EG)*F*(KG*CE2)
134             DQCD1Z=DCA1DZ*QA+DQCE1Z
135             DQCD2Z=DQCE2Z
136             DCA1  =DCA1DZ*DZ
137             DQFCE1=DQCE1Z*DZ
138             DQFCE2=DQCE2Z*DZ
139             DQDCD1=DQCD1Z*DZ
140             DQDCD2=DQCD2Z*DZ
141             CA1=CA1+DCA1
142             CE1=CE1+DQFCE1/QF
143             CE2=CE2+DQFCE2/QF
144             CD1=CD1+DQDCD1/QD
145             CD2=CD2+DQDCD2/QD
146       C     IF(INT(H/5)*5.NE.H)GO TO 22
147       C     PRINT *,CA1,CE1,CE2,CD1,CD2,
148             SMF=QA*CA1+QF*CE1+QF*CE2+QD*CD1+QD*CD2
149             WMF=QF+QD
150             SOLIDS=100.*SMF/(SMF+WMF)
151       C
152       C  Calculate solids masses
153       C
154             MCA1=AC*EG*DZ*CA1
155             MCE1=AC*(1.-EG)*F*DZ*CE1
156             MCE2=AC*(1.-EG)*F*DZ*CE2
157             MCD1=AC*(1.-EG)*(1.-F)*DZ*CD1
158             MCD2=AC*(1.-EG)*(1.-F)*DZ*CD2
159       C     IF(INT(H/5)*5.NE.H)GO TO 22
160       C     PRINT *, MCA1,MCE1,MCE2,MCD1,MCD2
161       C
162       C  Calculate grades and percent solids
163       C
164             GRADE=100.*(MCA1+MCD1+MCE1)/(MCA1+MCE1+MCE2+MCD1+MCD2)
165             ASH=100.*(MCE2+MCD2)/(MCA1+MCE1+MCE2+MCD1+MCD2)
166             SOL=MCA1+MCE1+MCE2+MCD1+MCD2
167             C1WTR=(1.-EG)*((1.-F)*CD1+F*CE1)
168             C2WTR=(1.-EG)*((1.-F)*CD2+F*CE2)
169       C     H2O=AC*(1.-EG)*DZ*(1.-(CE1+CD1+CD2+CE2))
170             H2O=AC*(1.-EG)*DZ*(1.-(C1WTR+C2WTR))
171             PSOL=100.*SOL/(SOL+H2O)
172          22 EG0=EG
173             DELTA0=DEL
174             R0=R
```

```
175      C
176      C  Write results
177      C
178            IF(INT(H/5)*5.NE.H)GO TO 10
179      C     PRINT *,H,ASH,GRADE,SOLIDS,PSOL,EG,DEL,QD,CD
180            WRITE(6,35)H,ASH,GRADE,PSOL
181         10 CONTINUE
182         35 FORMAT( T16,F5.1,T30,F5.2,T42,F5.2,T54,F5.2)
183            STOP
184            END
```

| HEIGHT FROM INTERFACE | PRODUCT ASH | PRODUCT GRADE | PERCENT SOLIDS |
|---|---|---|---|
| 0.0 | 13.44 | 86.56 | 20.77 |
| 5.0 | 13.02 | 86.98 | 20.88 |
| 10.0 | 12.62 | 87.38 | 20.98 |
| 15.0 | 12.25 | 87.75 | 21.07 |
| 20.0 | 11.89 | 88.11 | 21.15 |
| 25.0 | 11.54 | 88.46 | 21.23 |
| 30.0 | 11.21 | 88.79 | 21.30 |
| 35.0 | 10.90 | 89.10 | 21.36 |
| 40.0 | 10.59 | 89.41 | 21.43 |
| 45.0 | 10.30 | 89.70 | 21.48 |
| 50.0 | 10.02 | 89.98 | 21.53 |
| 55.0 | 9.75 | 90.25 | 21.58 |
| 60.0 | 9.50 | 90.50 | 21.63 |
| 65.0 | 9.25 | 90.75 | 21.67 |
| 70.0 | 9.00 | 91.00 | 21.71 |